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PHYSICAL REVIEW LETTERS

SECTION 2

ABSTRACTS OF ARTICLES TO BE PUBLISHED IN THE PHYSICAL REVIEW

In this section are printed the abstracts of Articles that have been forwarded to The American Institute of Physics for publication in The Physical Review. In quoting information obtained from this section before the appearance of the corresponding Article, reference should be made to "Physical Review (to be published)" rather than to this Journal.

ATTRACTIVE TWO-BODY INTERACTIONS IN PARTIALLY IONIZED PLASMAS. Gilda M. Harris, Lawrence Radiation Laboratory, University of California, Livermore, California (Received September 18, 1961; revised manuscript received October 24, 1961).

It has recently been shown that in a classical plasma a shielded Coulomb (Yukawa) potential describes the effective two-particle interactions through chains of intermediate particles. It therefore seemed reasonable to use this potential to approximate two-particle quantum interactions in a sea of charged particles, such as a hydrogen plasma. Consequently, an approximate solution of the Schrödinger equation was obtained for a hydrogen atom in a shielded Coulomb potential. With this potential, the number of H-atom bound states is finite and the energy eigenvalues are a function of the density and temperature. A variational calculation was performed using hydrogen-like single electron functions as a basis set and the effective nuclear charge as a variational parameter. Bound-state energies were obtained for 45 states of hydrogen in a grid of values for the screening constant. As the screening increases, the bound-state energy increases. For each state there is a maximum shielding above which the state is free, i.e., for which $E > 0$. According to the present calculation, the energy of the ground state becomes zero at a screening of 1.15 atomic units. Above this value then, no bound states can exist.

FAST NEUTRAL PARTICLES FROM ARC CATHODE. A. von Engel and K. W. Arnold, The Clarendon Laboratory, University of Oxford, Oxford, England (Received September 18, 1961).

The rotating drum method was used to measure the velocity distribution in the vapor jet which emerges from the cathode region in a magnetically stabilized high-vacuum arc discharge maintained between cooled copper electrodes. To obtain sufficient detection sensitivity part of the cathode was made of radioactive copper. The number distribution of copper atoms deposited on the drum as a function of their velocity perpendicular to its surface was measured by a Geiger-Müller counter. An average of about 4×10^4 cm/sec was found for the atomic beam velocity. It is suggested that misinterpretation of earlier observations has lead to apparent velocities of up to 10^6 cm/sec.

TEMPERATURE DEPENDENCE OF THE SURFACE TENSION OF He II. A. D. Singh, Department of Physics, University of Delhi, Delhi, India (Received September 1, 1961).

It is shown that the temperature dependence of the surface tension of liquid helium II can be understood on the ideal gas model for the liquid also, provided we re-enumerate the eigenfunctions in a bounded continuum so as to take into account the surface term. The maximum deviation of our results from the experimental ones of Allen and Misener is about seven percent, and from the theoretical calculations of Atkins only about two percent.

OBSERVATION OF TORQUE EXERTED BY PURE SUPERFLOW. Thomas R. Koehler* and John R. Pellam, California Institute of Technology, Pasadena, California (Received September 27, 1961).

Pure superflow is observed to exert a torque in flowing about an object of arbitrary shape. The magnitude of such torque agrees in every respect with expectations based on pure potential flow theory. The present results indicating pure potential flow for superfluid are considered to constitute

the complementary experiment to the earlier Craig-Pellam investigation of zero lift for superflow. We consider that these two experiments taken together establish completely the pure potential flow property for superfluid by verifying that (a) zero force, but (b) a prescribed torque are exerted upon an object of arbitrary shape and orientation. As a by-product to the present investigation, an independent evaluation of ρ_s/ρ for liquid helium was obtained in conformity with earlier measurements.

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APPROACH TO EQUILIBRIUM OF ELECTRONS, PLASMONS, AND PHONONS IN QUANTUM AND CLASSICAL PLASMA. David Pines and J. Robert Schrieffer, Department of Physics, University of Illinois, Urbana, Illinois (Received August 4, 1961).

The approach to equilibrium of electrons, plasmons, and phonons in finite temperature plasmas is studied in the random phase approximation. It is first shown that for an electron plasma in equilibrium, the long-wavelength, well-defined plasmons contribute a term to the free energy which is appropriate to a collection of independent bosons. In order to study nonequilibrium processes, an explicit plasmon distribution function is introduced. The matrix element for plasmon-electron coupling is calculated in the random phase approximation, and second-order perturbation theory is used to write down the equations which couple the electron and plasmon distribution functions. Equilibrium is shown to result from the competition between the spontaneous emission of plasma waves by single fast electrons and the Landau damping of the plasma waves due to the same group of particles.

The equation for the time rate of change of the electron distribution function reduces, in the classical limit, to a Fokker-Planck equation in which there appear diffusion and friction terms associated with plasma waves, of the type first considered by Klimontovitch. When an initial arbitrary nonequilibrium electron distribution is considered, it is seen that a plasma wave instability corresponds to coherent excitation of plasma waves by the electrons in contrast to the incoherent excitation associated with spontaneous emission.

The method is generalized to two-component electron-ion plasmas in which well-defined acoustic plasma waves exist, by introducing an explicit distribution function for the phonons (the acoustic plasmons). The approach to equilibrium and the two-stream instability are derived and discussed for the coupled electron-phonon system; results similar to those obtained for the electron-plasmon system are found.

THEORY OF SUPERCONDUCTIVITY. S. H. Liu, Research Center, International Business Machines Corporation, Yorktown Heights, New York (Received September 20, 1961).

Recently Eliashberg derived the theory of superconductivity by the Green's function technique. The energy gap equation was deduced by a self-consistent calculation. The result is similar to the integral equation of Bardeen, Cooper, and Schrieffer except that the effective electron interaction potential is different from the Bardeen-Pines potential as well as the Bogoliubov potential. The purpose of this paper is to discuss these differences. It shows that the "dangerous terms" in the Bogoliubov theory may be calculated in a different manner. The compensation of these dangerous terms leads to the result of Eliashberg. Therefore the self-consistent calculation of Eliashberg is but a different interpretation of the Bogoliubov's idea. However, it is argued that the most reliable way of treating the problem should be the variational method because it gives the lowest ground-state energy. This method gives all the results of Bogoliubov in the lowest order of approximation but is free from arbitrary interpretations.

MAGNETOACOUSTIC EFFECTS IN TILTED MAGNETIC FIELDS. Harold N. Spector, Institute for the Study of Metals and Department of Physics, University of Chicago, Chicago, Illinois (Received July 3, 1961; revised manuscript received October 11, 1961).

We have shown that, for the cases of a free-electron gas, in the two-spherical-band model, and the model of majority and minority carriers, certain portions of the Fermi surface can be mapped in detail. This can be done by using geometric resonances in the sound attenuation in

tilted magnetic fields, and the drift velocity of the carriers along the magnetic field can simultaneously be determined. For this to be possible, $\omega\tau$ must exceed unity and the Fermi velocity must not exceed the sound velocity by more than a factor of 100. Then the diameters of all the orbits, not merely the extremal orbits, can be measured and the drift velocity along the magnetic field determined as well. The general features of the phenomena considered do not prove dependent on the particular models used in our calculations.

In addition to the results specifically pertaining to tilted fields, we have found that when the assumptions of equal effective masses and relaxation times are dropped for a two-band model of a semimetal, the contribution of the two types of carriers to the ultrasonic absorption is additive. On examining the contribution to the absorption for a model of majority and minority carriers, we have found, also, that the minority carriers dominate the attenuation when they are in the region of geometric resonances.

SECONDARY ELECTRON EMISSIONS FROM METAL SURFACE BY HIGH-ENERGY ION AND NEUTRAL ATOM BOMBARDMENTS. S. N. Ghosh and S. P. Khare, J. K. Institute of Applied Physics, University of Allahabad, Allahabad, India (Received December 13, 1960; revised manuscript received July 29, 1961).

To obtain γ_e for H_2^+ ion bombardment on metal targets, an expression for the secondary electron emission coefficient by fast positive-ion bombardment is obtained after adopting a method of calculation similar to that employed by Sternglass. It is shown that the experimental values of γ_e for H_2^+ ion bombardment agree with the calculated values provided it is assumed that inside the target a hydrogen molecular ion is dissociated into a proton and a hydrogen atom, each having half the energy of the molecular ion. The dissociation cross section of hydrogen molecular ion inside the target may be given by $\sigma_d = K\sqrt{T}$, where $K = 1.2$, σ_d is expressed in units of 10^{-17} cm^2, and T is in Mev.

Secondary electron emission coefficient γ_e for high-energy hydrogen atom bombardment on a metal surface is also calculated and compared with experimental data. In the calculation, a neutral beam of hydrogen atoms is treated inside the target as composed of protons and electrons in addition to hydrogen atoms. Each of these three kinds of particles is capable of producing internal secondaries.

A fair agreement between the calculated and observed values of γ_e for H^+, D^+, H_2^+, and H^0 bombardments has been obtained.

PHONON-ASSISTED TUNNELLING IN SILICON AND GERMANIUM ESAKI JUNCTIONS. A. G. Chynoweth, R. A. Logan, and D. E. Thomas, Bell Telephone Laboratories, Murray Hill, New Jersey (Received September 27, 1961).

Phonon cooperation in indirect tunnelling transitions in narrow p-n junctions causes structure in the current voltage characteristics at low temperatures. These effects have been studied in silicon and germanium Esaki junctions using special equipment which plots continuously on an X-Y recorder the bias dependence of the current I, conductance $G = (dI/dV)$, and $(d^2I/dV^2)G^{-2}$. Twelve phonon and phonon-combination energies have been definitely revealed on the Si junctions and seven in the Ge junctions. Assignments of these energies are given; those for the Si junctions are mostly combinations of the transverse acoustic or optic phonons with intervalley scattering phonons and optic phonons of zero wave number. In the silicon junctions, in addition to the phonon-induced structure, over certain ranges of forward bias there are prominent oscillations in the second derivative curves. The origin of these oscillations is not satisfactorily understood though they have been tentatively ascribed to the Stark splitting of the energy bands in the high electric field of the junction.

DE HAAS–VAN ALPHEN EFFECT IN BISMUTH-TELLURIUM ALLOYS. Daniel Weiner,* Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received June 2, 1961).

De Haas–van Alphen measurements have been made on pure bismuth and several bismuth-tellurium alloys. It is found that the observed variation of extremal cross section and cyclotron effective mass with tellurium concentration and magnetic field orientation can be interpreted using a special case of Cohen's nonellipsoidal model of bismuth. The results indicate that there is a thermal energy gap between the conduction and valence band of about 0.046 ev in agreement

with various optical experiments and that there are six electron "ellipsoids." The results also agree with a model for the hole band involving one light-hole ellipsoid and one heavy-hole ellipsoid and are used as evidence against some other possible models for the hole band.

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LATTICE ANHARMONICITY AND OPTICAL ABSORPTION IN POLAR CRYSTALS. III. QUANTUM MECHANICAL TREATMENT IN THE LINEAR APPROXIMATION. R. F. Wallis, U. S. Naval Research Laboratory, Washington, D. C., and A. A. Maradudin, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received September 21, 1961).

A quantum mechanical calculation is given for the effect of lattice anharmonicity on the optical constants of an ionic crystal in the reststrahl region of the spectrum. A modification of the Kubo formalism for treating transport coefficients is employed. Only cubic anharmonic terms in the Hamiltonian are considered. In agreement with the correspondence principle, the present results in the limit of high temperatures reduce to those of the classical treatment which was given in a previous paper. A comparison is made between the present calculation and that of Born and Huang. The available experimental data are discussed.

RECOVERY OF ELECTRICAL RESISTIVITY IN GOLD QUENCHED FROM BELOW 700°C. M. Meshii, T. Mori,* and J. W. Kauffman, Department of Materials Science, Northwestern University, Evanston, Illinois (Received January 3, 1961; revised manuscript received November 3, 1961).

The residual electrical resistivity as measured at liquid helium temperature was determined as a function of annealing time at 62.4°C, for high-purity gold quenched from temperatures between 700° and 400°C at the rate of 25000°C/sec. For quenches begun from below 500°C an initial increase in resistivity occurs, followed by the usual decrease with aging time. The time to reach maximum resistivity is found to be a function of the quenched-in resistivity. The increase in resistivity may be due to vacancy clustering during the aging process. Further study is required to determine the role of impurity atoms in this proces For quenches above 500°C, some clusters apparently form during the quenching process and grow acting as stable vacancy sinks during subsequent aging; hence no increase occurs for these quench A calculation of the formation of divacancies and trivacancies during the annealing process, neglecting interactions with impurities, show appreciable clustering upon annealing following a low-temperature quench for binding energies of 0.3 ev. A maximum in the calculated resistivity due to formation of clusters is reached at the same order of magnitude of time as the maximum in resistivity is observed, and is followed by an approximately exponential decay which also corresponds to the experimental observations.

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NUCLEAR MAGNETIC DIPOLE COUPLING IN Al_2O_3. A. H. Silver, T. Kushida, and J. Lambe, Scientific Laboratory, Ford Motor Company, Dearborn, Michigan (Received July 13, 1961).

An investigation of the five nuclear magnetic resonance lines in Al_2O_3 shows a partially resolved structure for each of the five lines. This structure can properly be described in terms of a two-spin Hamiltonian with dipole-dipole coupling. The dipole coupling constant d is 0.3998 kc/sec and the linewidths are approximately $10d$. The analysis of the line shapes shows that the sign of the quadrupole coupling constant eqQ is positive.

NUCLEAR RESONANCE OF ALUMINUM IN SYNTHETIC RUBY. C. M. Verber, H. P. Mahon, and W. H. Tanttila, University of Colorado, Boulder, Colorado (Received September 11, 1961; revised manuscript received November 6, 1961).

The linewidth, line shape, and spin-lattice relaxation time T_1 have been measured as a function of temperature in single crystals of $(Al^{27})_2O_3$ having nominal Cr_2O_3 concentrations of 0% and 0.01, 0.1, 1.5, and 4% by weight. Of the two important contributions to the linewidth, the nuclear spin-spin interaction is 7 gauss and temperature independent. The contribution from the paramagnetic ions increased from zero at 300°K to as much as 42 gauss depending upon concentration,

temperature, and static magnetic field. An anomalous structure in the aluminum resonance was evident in both the steady-state and pulsed nuclear induction data. The $\frac{1}{2}$ to $-\frac{1}{2}$ transition has a splitting of 2.84 gauss with a $3\cos^2\theta-1$ dependence on the angle between the c axis and H_0. The splitting is independent of the observation frequency and chromium ion concentration. T_1 was observed to vary from 150 sec to 1.5 msec as a function of ion concentration and temperature and to vary in a manner not correctly given by existing theory. Many of the samples were observed to have a nonunique T_1.

OPTICAL SPECTRUM OF THE SEMICONDUCTOR SURFACE STATES FROM FRUSTRATED TOTAL INTERNAL REFLECTIONS. N. J. Harrick, Philips Laboratories, Irvington-on-Hudson, New York (Received August 28, 1961).

Through the use of multiple internal reflection and a modulation of the charge in the surface via the field effect, sufficient sensitivity is obtained to observe absorption of infrared radiation resulting from transitions involving the semiconductor surface states. This offers the possibility of obtaining the optical spectra and thus some rather direct information about the surface states. The principal other mechanism which contributes to the signal in this experiment is the free-carrier absorption arising from a modulation of the free-carrier density in the semiconductor space-charge region. The results of some observations on a silicon surface are discussed.

VACANCIES AND COMPLEXES IN THE NOBLE METALS. C. P. Flynn, Department of Physics, University of Illinois, Urbana, Illinois (Received August 28, 1961).

In Part I, the changes in resistivity of a noble metal on the introduction of vacancies into the lattice are calculated, taking into account the relaxation of the lattice around the defect site. The agreement between calculated and experimental resistivities is satisfactory for a particular model of the scattering power of the vacancy. Using this same model, it is then shown how the calculations of Fumi and Brooks concerning vacancy formation energies can be modified in the light of known surface energy data to yield quite reasonable predictions of formation energies of relaxed vacancies in noble metal lattices.

The agreement between calculated and experimental resistivity data implies that the phase shift of electron waves at the Fermi surface are accurate and this in turn permits the prediction of the electron density in the neighborhood of the vacancy. In Part II, this electron density is used in a treatment of the interaction energy between vacancies and substitutional impurities in the noble metals. The binding energies deduced in this way from scattering theory exhibit a strong dependence on the valence of the solute atom and are invariably positive for positive-valence solutes at nearest neighboring lattice sites to a vacancy. The computed binding energies are quantitatively compatible with the available experimental data.

The long-range characteristics of interaction between vacancies and both point and extended defects are also investigated in Part II. Particular attention is drawn (a) to the oscillatory nature of the interaction as a function of the separation of the defects, and (b) to the different dependence on this separation for interactions concerning the different types of defect.

Thus, in every case, the interaction energy oscillates with period k_F/π, k_F being the Fermi energy, while the decay of amplitude with separation takes the form r^{-3}, $r^{-5/2}$, or r^{-2} contingent upon the vacancy interacting with a second point defect, a dislocation, or a crystal boundary, respectively.

PARAMAGNETIC RESONANCE WIDTH IN IRON AND NICKEL. B. R. Cooper and F. Keffer,* Department of Physics, University of California, Berkeley, California (Received September 18, 1961).

A theory is presented to account for the unusually large linewidths found by Rodbell in the microwave resonance absorption of iron and nickel single crystals above the Curie points. The perturbing spin coupling is assumed to contain pseudodipolar (D) plus quadrupolar (Q) components. By the method of moments it is shown that the paramagnetic linewidth is proportional to $(D^2+bQ^2)/J$, where J is the exchange integral and $b=0$ for $S=\frac{1}{2}$, and $b=\frac{1}{27}$ for $S=1$. The ferromagnetic anisotropy constant at 0°K is proportional to $(-D^2/J)+eQ$, where $e=0$ for $S=\frac{1}{2}$, $e\sim 2$ to 5 for $S=1$. Since D, Q, and J are probably not very temperature sensitive, measurements of paramagnetic linewidth and ferromagnetic anisotropy,

together with well-known techniques for estimating J, may be used to deduce values of both D and Q. For iron and nickel it is shown that the contribution of Q to the linewidth is negligible, whereas the contribution from D is enormous, thus accounting for Rodbell's results.

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THERMAL CONDUCTIVITY OF NORMAL AND SUPERCONDUCTING ALUMINUM. C. B. Satterthwaite,* Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received September 26, 1961).

The thermal conductivity of three specimens of aluminum differing widely in residual resistivity was measured in the normal and superconducting states. The experimental method yielded quite accurate results for the ratio of the superconducting state thermal conductivity to that in the normal state, κ_S/κ_n. The ratios κ_S/κ_n, for specimens differing in electronic mean free path by as much as a factor of 140, were essentially the same over the range of temperature covered by the experiments, 0.32 to 1.20°K. Agreement of the experimental results with the theoretical predictions of Bardeen, Rickayzen, and Tewordt was quite good.

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LOW-TEMPERATURE LENGTH CHANGE MEASUREMENTS OF ELECTRON-IRRADIATED GERMANIUM AND SILICON. Frederick L. Vook, Sandia Laboratory, Albuquerque, New Mexico (Received September 11, 1961).

Precision measurements of the change in length of high-purity germanium and silicon were made upon 2-Mev electron irradiation and annealing. Two germanium samples were irradiated at maximum temperatures of 365°K and 86°K, respectively. The first sample was irradiated at 33 μa/cm^2 to an integrated flux of 5.4×10^{19} electrons/cm^2. The specific length expansion obtained was $\Delta L/L=(6.2\pm12.2)\times10^{-26}$ per 2-Mev electron/cm^2. The 86°K sample was irradiated at 10 μa/cm^2 to an integrated flux of 3.0×10^{19} electrons/cm^2. The specific length expansion was $(1.5\pm3.9)\times10^{-25}$ per 2-Mev electron/cm^2. Using the Seitz-Koehler simple theory of displacement ($E_d=30$ ev) and assuming no annealing at the low temperature, we calculate a concentration of displaced pairs equal to 1.6×10^{-3} with a corresponding fractional atomic volume change per interstitial-vacancy of $(f_i+f_v)_{Ge}=0.008\pm0.021$. The silicon sample was initially irradiated at ~50°K to an integrated flux of 8.3×10^{18} electrons/cm^2 and further irradiated at ~115°K to a total integrated flux of 7.9×10^{19} electrons/cm^2 and then annealed to room temperature. The specific length change for both bombardments was $\Delta L/L=(3.8\pm19.4)\times10^{-26}$ per 2-Mev electron/cm^2. The simple theory ($E_d=28$ ev) predicts a concentration of displaced pairs of 4×10^{-3} with a corresponding volume change per interstitial-vacancy of $(f_i+f_v)_{Si}=0.002\pm0.011$. These values are much smaller than would be expected from previous results for deuteron and neutron irradiations of germanium. Calculations of defect cluster size distributions for electron, deuteron, and neutron irradiations of germanium are given and combined with the experimental results to indicate that the volume change per defect is nonlinear and increases as the cluster size increases.

MEASUREMENT OF EQUILIBRIUM CONCENTRATIONS OF LATTICE VACANCIES IN GOLD. R. O. Simmons and R. W. Balluffi, University of Illinois, Urbana, Illinois (Received September 11, 1961).

The linear thermal expansion of a 99.999% gold bar has been measured throughout the range 15 to 1057°C by direct observation of the length expansion, $\Delta L/L$, with filar micrometer microscopes and by measurement of the lattice parameter expansion, $\Delta a/a$, by x-ray diffraction with a rotating-single-crystal method. The expansions agree within the experimental precision of about 1:10^5 at the lower temperatures. However, the values of thermal expansion obtained are about 1.5% larger than those in the literature. At the higher temperatures ($\Delta L/L-\Delta a/a$) becomes positive, providing that thermally generated defects are formed which are predominantly vacant lattice sites. The net added concentration of substitutional atomic sites, $\Delta N/N=3(\Delta L/L-\Delta a/a)$, just below melting temperature, is $(7.2\pm0.6)\times10^{-4}$ as obtained by an extrapolation of the data of only 3°C. $\Delta N/N$ can be described by $\exp(1.0)\times\exp(-0.94\text{ ev}/kT)$. These results are independent of any aggregation of the defects and of any lattice dilatation about the individual defects. At

the melting temperature, more than 80% of the defects are single vacancies if the divacancy and trivacancy binding energies are less than 0.4 ev and 1.0 ev, respectively.

The present concentrations are about 40% larger than those reported by DeSorbo from a calorimetric study of quenched foil. A critical assessment of the relationship between the present equilibrium measurements and results obtained by quenching methods is attempted. Combination of present $\Delta N/N$ values with $\Delta\rho$ and $(\Delta L/L)/\Delta\rho$ values of Bauerle and Koehler and with $\Delta L/L$ values of Takamura on quenched and annealed wires gives an electrical resistivity $\rho_v = 1.5 \pm 0.3$ μohm-cm/atomic percent vacancies and a volume expansion $\chi = 0.45 \pm 0.10$ atomic volume/vacancy.

THEORY OF THE ANOMALOUS PHOTOVOLTAIC EFFECT OF ZnS. Gertrude F. Neumark, Philips Laboratories, Irvington-on-Hudson, New York (Received July 31, 1961; revised manuscript received October 17, 1961).

A theory to account for anomalies observed in the photovoltage of faulted ZnS crystals is proposed. The theory assumes that the spontaneous polarization of hexagonal ZnS causes internal fields in these crystals, with opposing fields in cubic and in hexagonal material. If one then considers a unit of one cubic and one hexagonal segment, these fields will cause differences in carrier concentration, and resultant differences in photovoltage at the two band-gap barriers in such a unit. The net photovoltages of such units will be additive along the crystal. It appears that the proposed theory can account for observed photovoltages, both as to their magnitude and their reversal of sign with wavelength.

SPECIFIC HEAT OF FERRITES AT LIQUID HELIUM TEMPERATURES. S. R. Pollack and K. R. Atkins, Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania (Received September 18, 1961).

The specific heat of lithium, cobalt, magnesium, and nickel ferrite has been measured from 1.8°K to 5°K using a calorimetric technique similar to that used by Clement. A "heat switch" was used to cool the samples rather than a helium exchange gas thereby avoiding helium desorption effects. In all cases the specific heat C could be described by the relation $C = \alpha_M T^{3/2} + \beta T^3$, where T is the temperature and α_M and β are constants. The Debye temperatures were computed from β and were in good agreement with values obtained at liquid nitrogen temperatures. The $T^{3/2}$ temperature dependence verifies the spin-wave theory for these compounds although the values of α_M were consistently larger than anticipated from observed values of the Curie temperatures. Possible explanations for this are discussed. An extra term proportional to T^{-2} was observed for cobalt ferrite and this was identified as the nuclear contribution to the specific heat. From the magnitude of this term the magnetic field at the nucleus of the cobalt ion was evaluated and found to be approximately 410 koe.

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COVALENT BONDING AND CHARGE DENSITY IN DIAMOND. L. Kleinman,* Institute for the Study of Metals, University of Chicago, Chicago, Illinois, and J. C. Phillips, Department of Physics and Institute for the Study of Metals, University of Chicago, Chicago, Illinois (Received July 17, 1961; revised manuscript received September 11, 1961).

Recently Göttlicher and Wölfel have measured x-ray scattering factors from diamond powder that differ appreciably from the older values of Brill. Because scattering from the 1s core is small, information about the crystal covalent bond can be obtained. In the Brillouin zone theory of crystal wave functions the valence charge density varies throughout the valence band. Free-atom approximations to the diamond charge density replace the sp^3 crystal wave functions at $\vec{k} = 0$ by atomic sp^3 charge densities. This approximation is insufficient to explain covalent bonding effects, such as the "forbidden" reflection F_{222}.

We have examined crystal charge densities throughout the valence band. We find that there are two important mechanisms which affect the valence charge density. The first is a linear response to the ionic potentials. This may be called dielectric screening; it is dominant in metals, and it gives no contribution to F_{222}. The second is nonlinear in the ionic potentials, and may be called crystal hybridization; it is responsible for covalent effects in the charge density, and it makes no contribution to F_{220}. The contribu-

tions of both mechanisms to F_{111}, F_{220}, F_{222}, F_{311}, and F_{400} have been computed. Surprisingly the results are in good agreement with the older data of Brill and simple models of the covalent bond, while significant disagreement with Göttlicher and Wölfel's measurements is found.

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HARDNESS OF NONMETALLIC SOLIDS ON AN ATOMIC BASIS. Johannes N. Plendl and Peter J. Gielisse, Air Force Cambridge Research Laboratories, Office of Aerospace Research, Bedford, Massachusetts (Received July 10, 1961).

Hardness values of 65 nonmetallic crystalline solids of 16 different structures are compared with data of a newly introduced concept: "the volumetric lattice energy." When such solids are classified according to a new interpretation of lattice anharmonicity, a set of linear interrelations is obtained which covers the entire hardness range. Thus, on the basis of interatomic cohesive forces, the over-all hardness of these solids becomes unambiguously defined. Hardness receives the dimension (ergs/cm^3) or (kcal/cm^3), and an absolute scale. Such hardness data become independent of anisotropy, structure type, and valency of the atoms. Conversion from relative to absolute hardness, as well as estimation of lattice energy data through appropriate hardness testing, becomes possible. For example, the extremely high-hardness value of the rare type II diamond could be determined to be 1.7 times that of the type I diamond, purely from atomic data. Analogous treatment of the hardness of metallic solids poses additional problems.

INFLUENCE OF DISLOCATION ON DIFFUSION RATE OF *F* CENTERS IN KCl. Hiroyuki Mizuno and Shigeko Moyamoto, Matsushita Electronics Corporation, Takatsuki, Osaka, Japan (Received August 15, 1961).

An experimental confirmation of the *F*-center diffusion along disordered interfaces in KCl single crystals is given. KCl single crystals were exposed to potassium vapor after (a) they had been bent around the [100] crystal axis yielding only one slip system, or (b) had been compressed along a [100] direction yielding a single pair of mutually perpendicular slip systems. The crystal showed a distinct anisotropy of *F*-center diffusion. The explanation for this anisotropy can be based on the crystal dislocations.

CYCLOTRON RESONANCE IN ALUMINUM. T. W. Moore* and F. W. Spong, Department of Physics, University of California, Berkeley, California (Received September 25, 1961).

Preliminary results of cyclotron resonance experiments at 35 kMc/sec in aluminum having a relatively long relaxation time at 4.2°K ($\omega\tau \sim 20$) are presented. Discrepancies and errors in earlier cyclotron resonance experiments are resolved. A tentative orbit identification scheme is proposed which indicates (a) that the second zone Fermi surface is consistent with the single orthogonalized plane wave (OPW) model, and (b) that the third zone Fermi surface is connected and that the connecting region is thicker than in the single OPW model. The effective masses obtained are compared with the values predicted by the single OPW model and with other experimental values, where pertinent. Departures of the effective mass value of orbits in the second zone from values given by the single OPW model are noted and may be attributable to electron-electron or electron-phonon effects.

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FLUORESCENCE OF CUBIC ZnS:Cl CRYSTALS. H. Samelson and A. Lempicki, General Telephone & Electronics Laboratories, Inc., Bayside Laboratories, Bayside, New York (Received September 5, 1961).

A series of fluorescence spectra (between 10°K and 300°K) and thermoluminescence measurements have been performed on self-activated cubic ZnS crystals prepared by two different methods. The vacuum grown crystals (Type I) show two series of edge emission lines (*A* and *B*) and a broad band blue emission (*SA*). Thermoluminescence results indicate the presence of three traps. Crystals grown in H_2S-HCl atmosphere (Type II) show only the long-wavelength series of edge emission lines (*B*), the same blue emission (*SA*), and, at low temperature, a weaker broad band emission at an intermediate wavelength (*SAL*). The thermoluminescence results indicate the presence of only one trap.

Using only the defects known to be present (Cl substituted at S sites, S vacancies, and Zn vacancies) and the decay of excitons, a model is proposed to account for the observed luminescent processes.

EFFECT OF COMPOSITION ON THE SUPERCONDUCTING TRANSITION TEMPERATURE OF TANTALUM CARBIDE AND NIOBIUM CARBIDE. A. L. Giorgi, E. G. Szklarz, E. K. Storms, Allen L. Bowman, and B. T. Matthias,* Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received August 28, 1961).

Tantalum carbide and niobium carbide samples with carbon to metal molar ratios varying from 0.754 to 0.987 for tantalum carbide and from 0.70 to 0.977 for niobium carbide were examined for superconductivity. Both niobium and tantalum carbides were found to be superconducting with transition temperatures which vary with composition. The major conclusion is that the highest temperature is reached for the stoichiometric composition and any deviation therefrom results in a lowering and eventual disappearance (less than 1.05°K) of T_C.

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STABILITY OF ONE-DIMENSIONAL FERROMAGNETIC MICROSTRUCTURES. William Fuller Brown, Jr., Department of Electrical Engineering, University of Minnesota, Minneapolis, Minnesota, and S. Shtrikman,* The Franklin Institute Laboratories for Research and Development, Philadelphia, Pennsylvania (Received September 12, 1961).

The equilibrium equations for the distribution of magnetization directions in a homogeneous ferromagnetic body possess one-dimensional solutions with some resemblance to a domain structure. It is shown here that under strictly one-dimensional conditions, which are possible only in an infinite plane plate, all the solutions in which the magnetization is not uniform are unstable, and that the Bloch wall acquires its stability from three-dimensional considerations implicit in the initial formulation of the problem.

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MAGNETOACOUSTIC MEASUREMENTS IN SILVER AT 230 Mc/sec AND 4.2°K. Vernon J. Easterling and Henry V. Bohm, Department of Physics, Wayne State University, Detroit, Michigan (Received September 21, 1961).

Measurements of acoustic attenuation in silver have been made with 150-233 Mc/sec longitudinal sound waves in magnetic fields up to 15 000 oersteds and at a temperature of 4.2°K. Plots of the ultrasonic pulse height as a function of the reciprocal of the magnetic field strength show from ten to fifteen maxima and minima for several orientations. Numerical data are presented and discussed in some detail. The Pippard-type theoretical model of the silver Fermi surface is compared with our measurements.

NONPARABOLIC CONDUCTION BAND IN HgSe AND $HgSe_{0.5}Te_{0.5}$. G. B. Wright, A. J. Strauss, and T. C. Harman, Lincoln Laboratory, Massachusetts Institute of Technology, Lexington, Massachusetts (Received September 20, 1961).

The dependence of electron effective mass (m^*) on carrier concentration (N) has been determined for samples of n-type HgSe ($N = 5\times10^{17}$ to 3×10^{19} cm^{-3}) and $HgSe_{0.5}Te_{0.5}$ alloy ($N = 9\times10^{17}$ to 9×10^{18} cm^{-3}) by means of infrared reflectivity and magnetoreflectivity measurements. In both cases, m^* increases markedly with N. For HgSe, the variation of m^* with N is quantitatively consistent with the conduction band model derived by Kane for InSb. The band parameters derived from the data are $P = 8\times10^{-8}$ ev-cm, in close agreement with the values determined for InSb and other III-V compounds, and $\epsilon_g \sim 0.2$ ev. The data for $HgSe_{0.5}Te_{0.5}$ appear to be qualitatively consistent with Kane's model, although they exhibit too much scatter for quantitative comparison.

DIELECTRIC BREAKDOWN IN CADMIUM SULFIDE. Richard Williams, RCA Laboratories, Princeton, New Jersey (Received September 26, 1961).

The dielectric breakdown field strength for CdS has been measured by a technique employing blocking contacts on conducting crystals. Values obtained with different crystals are in the range 0.9×10^6 to 2.5×10^6 volts/cm. Experiments in which carriers are injected by intermittent illu-

mination show that impact ionization does not occur during breakdown. A variety of evidence is presented indicating that breakdown is due to internal field emission.

FERRIMAGNETIC STRUCTURE OF A MAGNETITE CRYSTAL AS REVEALED BY ELECTRON DIFFRACTION. S. Yamaguchi, Institute of Physical and Chemical Research, Tokyo, Japan (Received September 11, 1961).

The process of thermal perturbation in electron diffraction is used to find the direction of easy magnetization, [111], of the lattice of magnetite. In the diffraction pattern of magnetite with the incidence along the $[1\bar{1}0]$ direction, the thermomagnetic displacement of the diffraction spots takes place perpendicular to the [111] axis. This fact leads to the direction of the magnetization according to the Lorentz force.

MÖSSBAUER STUDY OF ISOMER SHIFT, QUADRUPOLE INTERACTION, AND HYPERFINE FIELD IN SEVERAL OXIDES CONTAINING Fe^{57}. G. Shirane and D. E. Cox, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania, and S. L. Ruby, Westinghouse Materials Laboratories, East Pittsburgh, Pennsylvania (Received September 5, 1961).

Mössbauer absorption by Fe^{57} was used to study isomer shift, quadrupole splitting, and hyperfine fields in several oxides containing iron. The established relations between isomer shifts and ionic states of Fe were further confirmed by measurements on $Fe^{2+}Ti_2O_5$, $Fe_2^{3+}TiO_5$, and several other mixed oxides. By utilizing this relation, in turn, to identify the ionic states when these are in doubt, it has been found that $(Fe, V)_2O_3$ incorporates the cation pair Fe^{3+}-V^{3+}, rather than Fe^{2+}-V^{4+}, and that "$SrFeO_3$" contains Fe^{4+} and Fe^{3+} in the ratio of 3:1. A well-defined quadrupole splitting was observed in $Fe_{1-x}O$ and in a number of the solid solutions (Fe, Mg)O, although their over-all symmetry is cubic. This is explained by the assumption of local asymmetry at the Fe nuclei caused either by vacancies or foreign ions. Hyperfine fields at the Fe^{3+} nuclei in $(Fe, Cr)_2O_3$, $(Fe, V)_2O_3$, and $(Fe, Al)_2O_3$ have values between 520 and 540 koe when extrapolated to 0°K, and the results are discussed in conjunction with their magnetic properties.

CLASSICAL MICROSCOPIC MODEL FOR MAGNETIC RESONANCE INCLUDING RELAXATION EFFECTS. R. K. Waring, Jr., Central Research Department, E. I. du Pont de Nemours and Company, Wilmington, Delaware (Received September 25, 1961).

A classical theoretical model for magnetic resonance is suggested, and several of its implications are developed. The solution to Bloch's equations for magnetic resonance, without relaxation, is assumed for each incremental portion of the total magnetization of a system. With a suitably rotating coordinate system, an investigation is then made of the steady-state angular distribution of magnetization that results from the random relaxation process to which each element of magnetization is subject. When the longitudinal relaxation time, T_1, is taken to equal the transverse relaxation time, T_2, this procedure leads to the same results as the direct solution of Bloch's equations with relaxation. In addition, the model yields a general description of the saturated dispersion signal for sinusoidal modulation of the external field, for both homogeneous and inhomogeneous broadening of the resonance line. A method for measuring T_1 based on this description is presented. For $T_1 \neq T_2$, the model reveals on classical, geometrical grounds an inconsistency in Bloch's implicit assumption that the transverse relaxation effects are independent of the applied radio-frequency field.

SPIN WAVES IN EXCHANGE-COUPLED COMPLEX MAGNETIC STRUCTURES AND NEUTRON SCATTERING. Albert W. Sáenz, U. S. Naval Research Laboratory, Washington, D. C. (Received May 1, 1961; revised manuscript received September 18, 1961).

In this paper, we develop a spin-wave theory of the Holstein-Primakoff type for exchange-coupled crystals with an arbitrary number n of magnetic ions per primitive magnetic unit cell, when the resultant electronic spin vectors of these ions are mutually parallel or antiparallel in a given domain, except for spin-wave fluctuations. A simple and systematic method is presented for finding a complete set of normal spin-wave modes. This method is used to derive a convenient formula for the component of the total electronic spin vector of the magnetic ions in a domain along the axis of spin alignment. When the magnetic ani-

sotropy and external magnetic field contributions vanish, we show the existence of at least one "acoustic" branch among the $\leq n$ distinct branches of the spin-wave spectrum. When all the $m \geq 1$ acoustic branches existing in the absence of these contributions are identical, we prove that the acoustic spin-wave energies corresponding to a given wave number $\vec{\kappa}$ are of $O(|\vec{\kappa}|^{2m})$ for $|\vec{\kappa}| \to 0$. The situation in which a single acoustic branch exists when no external magnetic field or anisotropy effects are present is studied in detail and, under suitable restrictions, an explicit formula is derived for the energies of the magnons of this branch for $|\vec{\kappa}| \to 0$. We apply this spin-wave theory to obtain general cross-section formulas for the one-magnon zero-phonon scattering of neutrons by the class of exchange-coupled crystals referred to in the first sentence of this abstract, when the magnetic ions in these crystals are completely quenched orbitally. The formulas in question are used to predict a spin-wave phenomenon of wide generality for polarized incident neutrons. This phenomenon is of particular experimental interest in connection with the acoustic spin-wave scattering of such neutrons by crystals of this class having a single acoustic branch and has been qualitatively confirmed by experiments on magnetite. For the last-mentioned crystals, we use an exact limit result of this paper to suggest a simple approximate form of the general cross-section equations pertaining to acoustic spin-wave scattering, when only magnons of sufficiently small $|\vec{\kappa}|$ are of interest.

SCATTERING OF SLOW NEUTRONS FROM PROPANE GAS. K. A. Strong, G. D. Marshall, R. M. Brugger, and P. D. Randolph, Atomic Energy Division, Phillips Petroleum Company, Idaho Falls, Idaho (Received September 18, 1961).

Measurements of the partial differential neutron scattering cross sections for room temperature propane gas are reported. These measurements were made at incident energies of 0.0101, 0.0254, 0.0736, and 0.102 ev at seven scattering angles between 16.3° and 84.7° using the Materials Testing Reactor phased-chopper velocity selector.

The data are converted to the scattering-law presentation and compared with three theoretical calculations: (a) The ideal gas, using an effective mass obtained from an average of the mass tensors for the three types of H atoms in propane, gives poor agreement. (b) The Krieger-Nelkin approximation, which includes the effect of zero-point vibrations gives limited agreement for energy transfer less than $0.5k_BT$ at intermediate momentum transfers. At large momentum transfers where vibrational effects become important it underestimates the cross section. (c) A modification of the Krieger-Nelkin theory that includes the effects of single-quantum transitions from the three lowest vibrational states gives better agreement. The discrepancies still present at large momentum and energy transfers are attributed to an uncertainty in the methyl-group barrier height for the three lowest energy modes, to the harmonic oscillator approximation for these modes, and to the approximate molecular orientation averaging used in the calculation.

SPECTRAL DIFFUSION DECAY IN SPIN RESONANCE EXPERIMENTS. J. R. Klauder and P. W. Anderson, Bell Telephone Laboratories, Murray Hill, New Jersey (Received September 1, 1961).

In spin resonance experiments random flipping by T_1 or T_2 processes of nearby, nonresonant spins introduces fluctuations into the precessional frequency of the observed spins. These fluctuations may be described by means of a stochastic model, and for wide classes of both Markoffian and non-Markoffian distributions we make predictions for the line shape, for the free induction decay, and for various spin-echo signals. If the homogeneous broadening of the line is due to a dipolar interaction term, then we find that the conditional distribution for the precessional frequency has the shape of a Lorentzian with a cut-off on the wings, rather than a Gaussian shape as commonly assumed. The causes and consequences of Lorentzian diffusion are analyzed in detail for samples in which T_1 processes control the source of local frequency fluctuations and for samples in which T_2 processes dominate. Recent two- and three-pulse spin-echo experiments of Mims et al. dramatically confirm the predictions of Lorentzian diffusion for electron paramagnetic resonances in samples with temperature-dependent diffusion, as well as with temperature-independent diffusion. "Instantaneous" diffusion caused by the action of the applied pulses is predicted by our model and explains features of Mims' data. The generality of our principal results still permits the outcome of various resonance experiments to be predicted, even when a simple dipolar interaction is no longer an adequate model.

FORMATION OF NO_2 BY CHARGE TRANSFER AT VERY LOW ENERGIES. R. K. Curran, Westinghouse Research Laboratories, Pittsburgh, Pennsylvania (Received September 25, 1961).

The formation of NO_2^- by charge transfer of various negative ions of near 0-ev kinetic energy in NO_2 has been observed. The explanation of the very large cross section observed for the charge transfer process may possibly be found in the proposal of Rapp and Magee for the enhancement of the charge transfer cross section for nonresonant systems at low energy through the influence of the polarization of the target molecule caused by the incoming ion. A lower bound of 3.82 ev is placed on the electron affinity of NO_2.

ALIGNMENT OF THE H_2^+ MOLECULAR ION BY SELECTIVE PHOTODISSOCIATION. I. H. G. Dehmelt and K. B. Jefferts, Department of Physics, University of Washington, Seattle, Washington (Received September 27, 1961).

On the basis of experimental results obtained by Linlor et al., Fischer, and in this laboratory, demonstrating photodissociation time constants as short as 0.1 sec and trapping times as long as several seconds as practical for the H_2^+ molecular ion, the feasibility is discussed theoretically of a novel technique to align trapped molecular ions and also monitor the alignment by selective photodissociation. First the limiting case of no electronic and nuclear spins and very large rotational quantum numbers is treated, based on the concept that the photodissociation rate R is proportional to the average square component of the electric light vector perpendicular to the axis of molecular rotation. This angular dependence of R, when a bunch of ions created by a short electron bombardment pulse is subsequently irradiated, causes the ions in certain magnetic sublevels to decay much more slowly than others. Consequently when conditions for the preservation of alignment are favorable, the sample becomes increasingly aligned. Furthermore, since a partially aligned sample photodissociates more slowly than an unaligned one, more molecular ions $\bar{N}$ will remain when alignment is allowed to develop than when it is continuously destroyed, resulting in only N ions. Next the nuclear and electronic spins are taken into account and numerical values for the dissociation rates of the 30 magnetic sublevels of the first three rotational states of the H_2^+ ion are evaluated for linear light polarization. One sees that on the basis of a sample of 10^9 H_2^+ ions decaying due to photodissociation to 2×10^7 ions, one might expect an optimum signal $(\bar{N}-N)/N \approx 0.25$ compared to a statistical uncertainty of about 0.0003 for two consecutive pulses. The possibilities inherent in the scheme to observe the rf spectrum of H_2^+ are pointed out.

EXPERIMENTS ON PAIR AND TRIPLET PRODUCTION BY 5-90 Mev X RAYS. E. H. Webb, H. S. Sandhu, R. C. Mohanty, and R. R. Roy, Nuclear Physics Laboratory, Department of Physics, The Pennsylvania State University, University Park, Pennsylvania (Received August 18, 1961; revised manuscript received September 25, 1961).

The energy partition between the positrons and electrons of 5787 pairs and 1872 triplets produced by a hardened beam of x rays of maximum energy 90 Mev, has been investigated using a nuclear emulsion technique. The results agree approximately with the theory of Bethe and Heitler. The relative total cross sections of triplet production agree, within experimental error, with the theory of Vortuba and Wheeler and Lamb with an exchange correction. Similarly, the angular distributions of both pairs and triplets can be explained within experimental error either with the theory of Borsellino or with that of Bethe.

INELASTIC SCATTERING BY DEFORMED NUCLEI. J. S. Blair, D. Sharp,* and L. Wilets, University of Washington, Seattle, Washington (Received September 13, 1961).

The techniques of the sharp and rounded cutoff models for elastic scattering have been extended to the calculation of nuclear monopole and quadrupole excitation in the adiabatic approximation. In the case of scattering from nuclei with quadrupole deformations, a spheroidal coordinate system is introduced where one of the coordinate ellipsoids coincides with the nuclear surface; the wave equation separates outside the range of the nuclear potential. Formal expressions and results of numerical calculations for the cross sections are presented for a case where simple assumptions are made about the functional form of the partial wave amplitudes and where only terms linear in nuclear deformation are retained. One limit of the form for the spherical partial

wave amplitudes leads to a sharp-cutoff model which goes over into the Fraunhofer results for moderate and large values of the critical angular momentum. It is found that graphs of the excitation cross section (and to a lesser extent the elastic scattering cross section) fall into a single-parameter family of "universal curves" when plotted against (scattering angles) × (critical angular momentum + $\frac{1}{2}$). The single parameter is (thickness of transition region in l spaces) ÷ (critical angular momentum). No detailed comparisons with experiment are made, but the model is capable of reproducing the qualitative results of alpha-particle scattering at energies well above the Coulomb barrier.

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DEUTERON STRIPPING BY HEAVY NUCLEI. W. D. Barfield, B. M. Bacon, and L. C. Biedenharn,* Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico (Received September 22, 1961).

Theoretical angular distributions for the proton groups following bombardment of Pb^{206} and Bi^{209} by deuterons with energies less than 10 Mev are compared with experimental measurements of Stokes. Both theoretical and experimental angular distributions show peaking in the backwards direction characteristic where Coulomb effects dominate the stripping. The theoretical curves fall off faster than the experimentally measured distributions, which show a tendency to level off in the region $\Theta > 140°$. The insensitiveness of the stripping integral to the value of the cutoff radius parameter is demonstrated by numerical calculations.

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ELASTIC SCATTERING IN THE N-N, C-N, AND C-C SYSTEMS. A. M. Smith and F. E. Steigert, Sloane Physics Laboratory, Yale University, New Haven, Connecticut (Received July 21, 1961).

Differential cross sections have been measured for the elastic scattering of N^{14} from C^{14}, N^{14} from N^{14}, and C^{12} from C^{12} to a relative precision of about 5%, and an absolute accuracy of 15%. Incident energies of nominally 10 Mev/amu were employed, and an angular range was covered from approximately 4° to 24° in the center-of-mass system. The use of solid-state detectors rendered energy resolution of the order of 1.5%. Theoretical cross sections calculated from both the simple-diffraction model and the sharp-cutoff model were fitted to the experimental data, giving interaction radii for the three reactions of 6.44, 6.68, and 6.23 fermis, respectively.

USE OF NORMAL COORDINATES IN NUCLEAR PHYSICS. F. J. Smith* and J. G. Brennan, Catholic University of America, Washington, D. C. (Received September 18, 1961).

It is shown that normal coordinates are not difficult to use in nuclear theory if it is assumed that the masses of the nucleons are equal. In the shell model with harmonic oscillator functions they have the advantage that they exclude the spurious states due to the motion of the center of mass.

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ELASTIC AND INELASTIC SCATTERING OF 28-Mev DEUTERONS. R. J. Slobodrian, Synchrocyclotron Laboratory, Nuclear Physics Division, Physics Department, Comisión Nacional de Energía Atómica, Buenos Aires, Argentina (Received January 10, 1961; revised manuscript received October 23, 1961).

Natural lithium, polyethylene, and aluminum targets were bombarded with 28-Mev deuterons. The outgoing particles were analyzed with a scintillation spectrometer. The angular distributions for the following processes were measured: $Li(d, d)Li$, $Li^7(d, d')Li^{7*}-4.61$ Mev, $C^{12}(d, d)C^{12}$, $C^{12}(d, d')C^{12*}-4.43$ Mev, $Al^{27}(d, d)Al^{27}$, $Al^{27}(d, d')Al^{27*}-2.21$ Mev, and $H^1(d, p)H^2$.

The $Li^7(d, d')Li^{7*}$ 4.61-Mev angular distribution can be fitted by superimposing plane-wave direct-reaction curves corresponding to $l=0$, $a=3.4$ f, and $l=2$, $a=3.9$ f, where a is the interaction radius and l the angular momentum change; this leads to an odd-parity assignment for the level. The angular distribution of the reaction $C^{12}(d, d')C^{12*}-4.43$ Mev is fitted by superimposing the curves of $l=0$, $a=4.1$ f, and $l=2$ with $a=4.6$ f. The $Al^{27}(d, d')Al^{27*}-2.21$-Mev angular distribution is adequately fitted with a curve of $l=1$, $a=5.1$ f; therefore there follows an odd-parity

assignment for the level. The possible spin assignments of the levels are discussed. The inelastic scattering curves yield some evidence of a total spin flip of the deuteron in the reaction.

The elastic scattering angular distributions show the usual diffraction pattern, and interaction radii were calculated using an optical analogy that implies the scattering of waves from a black disk. The angular distribution of the interaction $H^1(d,p)H^2$ is consistent with measurements performed at neighboring energies and it also agrees with the curve obtained with a Serber-type force.

MOMENTUM TRANSFER IN HEAVY-ION-INDUCED FISSION. Torbjørn Sikkeland, Eldon L. Haines, and Victor E. Viola, Jr., Lawrence Radiation Laboratory, University of California, Berkeley, California (Received September 7, 1961).

The forward linear momentum transfer in reactions leading to fission between heavy ions such as C^{12}, N^{14}, O^{16}, and Ne^{20} and the target nuclei Ho, Au, Bi, and U^{238} has been investigated by measuring the angular correlation between the fragments. The experimental values for the most probable parameters for center-of-mass transformation for these systems are compared with calculated values. For all the systems, the dominant reaction involves a full momentum transfer by the heavy ion to the fissioning nucleus. It is observed that for the system $Au+Ne^{20}$ and $Bi+Ne^{20}$, reactions with incomplete momentum deposition contribute 5.1% and 8.6%, respectively, to the total fission cross section. For U^{238} targets this admixture amounts to 12 to 17% for all ions at the highest bombarding energy and decreases with decreasing projectile energy. Possible reaction mechanisms leading to fission are suggested. A brief discussion of the method and its application is given.

NEUTRON GIANT RESONANCES—NUCLEAR RAMSAUER EFFECT. J. M. Peterson, Lawrence Radiation Laboratory, University of California, Livermore, California and Institute for Theoretical Physics, Copenhagen, Denmark (Recieved July 31, 1961).

Three continuous families of broad maxima and minima are observed in neutron total cross sections between 0.1 and 100 Mev. All shift smoothly to higher energy with increasing mass number. The relationships among the families, their energy-mass number dependence, and their detailed locations can be understood in terms of a semiclassical treatment of a simplified optical model. The oscillations are seen to result from interference between the part of the neutron wave which has traversed the nucleus with the part which has gone around. This nuclear situation is analogous to the Ramsauer effect in electron interactions with noble gases. An alternative explanation of the broad maxima as due to resonances of single partial waves is not valid because in general several partial waves are simultaneously important and because the partial wave characteristics change rapidly as one traverses a continuous family of maxima. The widths of the broad maxima are related more to the parameters of the real potential well than to the depth of the imaginary potential well.

STRAGGLING EFFECTS ON RESONANT YIELDS. H. W. Lewis,* Palmer Physical Laboratory, Princeton University, Princeton, New Jersey (Received September 20, 1961).

The finite nature of energy loss processes for a charged particle in a material leads to a "ringing" effect in the thick-target yield curve. The effect is described and evaluated.

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STUDY OF Zr^{92} LEVELS EXCITED IN THE (d,p) REACTION. H. J. Martin, Jr., H. B. Sampson, and R. L. Preston,* Indiana University, Bloomington, Indiana (Received July 27, 1961; revised manuscript received October 23, 1961).

Seven energy levels in Zr^{92} were excited by the (d,p) reaction using 10.85-Mev deuterons. These levels were at excitation energies of 0, 0.94, 1.49, 1.86, 2.06, 2.91, and 3.28 Mev. The last two levels appear to be complex. Angular distributions were measured for all of these levels except the one at 1.86-Mev excitation. The distributions for the three lowest levels were d-wave; the distributions for the three highest levels were s-wave. Cross-section comparisons were made with earlier work on Zr^{91} levels. These comparisons indicated that the dominant configuration of the lowest

levels is $(d_{5/2})^2$ and that the 2.91-Mev level is largely $(d_{5/2}s_{1/2})$. A calculation of the neutron-neutron interaction, using vacuum-like, singlet forces between the nucleons, gives fair agreement with the experimental results.

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DECAY OF Sb^{115}. M. L. Sehgal, Brookhaven National Laboratory, Upton, New York (Received May 31, 1961; revised manuscript received October 12, 1961).

Sb^{115} (32 min) decays by electron capture and positron emission to Sn^{115}, the positron branch accounting for $(12 \pm 3)\%$ of the disintegrations. Of the total disintegrations 94% lead directly to a previously known 499-kev level in Sn^{115}. The remaining 6% of the disintegrations are to an excited state in Sn^{115} and are followed by two gamma rays in cascade whose energies are 980 kev and 1240 kev, respectively. The crossover gamma ray of 2220 kev has also been seen, with an intensity equivalent to 1% of the disintegrations. The cascade and crossover gamma rays lead to the 499-kev level and coincidences have been observed between these gamma rays. An upper limit of 0.4% was obtained for a beta branch from In^{115m} to the 499-kev level in Sn^{115}. These levels in Sn^{115} seem to be too high in energy to correspond to the predicted low-lying levels in Sn^{115} of Kisslinger and Sorensen.

HIGH-RESOLUTION STUDY OF THE $B^{10}(He^3, p)C^{12}$ REACTION. C. P. Browne, W. E. Dorenbusch, and J. R. Erskine,* University of Notre Dame, Notre Dame, Indiana (Received September 26, 1961).

Proton groups from the $B^{10}(He^3, p)C^{12}$ reaction leading to levels up to 16.2-Mev excitation were analyzed with a broad-range spectrograph. At a bombarding energy of 3.74 Mev, spectra were taken at 11 angles from 5 to 139 degrees. Levels were found at excitation energies of 4.43(7), 7.655 $\pm$0.006, 9.645 $\pm$ 0.006, 10.849 $\pm$ 0.025, 11.841 $\pm$ 0.025, 12.713 $\pm$ 0.006, 13.29 $\pm$ 0.03, 14.083 $\pm$ 0.015, 15.108 $\pm$0.006, and 16.108 $\pm$ 0.006 Mev. Widths of the 9.6-, 10.8-, 11.8-, 13.3-, and 14.1-Mev levels are 35 $\pm$6, 320 $\pm$ 30, 245 $\pm$ 30, 430 $\pm$ 100, and 252 $\pm$ 15 kev, respectively. Widths of the other levels are less than 15 kev. No levels were found at excitations of 9.0, 11.1, or 15.6 Mev. A level 2 Mev wide at 10.1-Mev excitation would not have been seen above the high background from three-particle breakup. Yields at each angle were measured relative to the yield of the $B^{10}(He^3, d)C^{11}$ reaction. An angular distribution of this reaction was obtained. Absolute differential cross sections were found by comparing the yields with the known yield from the $B^{10}(d, p)B^{11}$ reaction. Cross sections for the $B^{10}(He^3, p)C^{12}$ reaction vary from 0.02 mb/sr to 0.6 mb/sr.

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POLARIZATION OF PROTONS IN $C^{12}(d,p)C^{13}$. R. G. Allas and F. B. Shull, Washington University, Saint Louis, Missouri (Received September 22, 1961).

The measurement of polarization of protons from the $C^{12}(d,p)C^{13}$ ground-state reaction has been continued and extended to larger angles. The new values obtained are -42% at 42.5° (lab); -35.4% at 46°; -51.5% at 52°; -22.5% at 61.5°; -20% at 64.5°; -6% at 70°; +35% at 76°. The axis of quantization is taken as $\vec{n} = \vec{k}_d \times \vec{k}_p$.

NUCLEAR STRUCTURE STUDIES IN THE ZIRCONIUM REGION WITH STRIPPING REACTIONS. B. L. Cohen, University of Pittsburgh, Pittsburgh, Pennsylvania (Received September 15, 1961).

The single quasi-particle levels in Sr^{89}, Zr^{91}, Zr^{93}, Zr^{95}, and Zr^{97} are located and identified by (d,p) and (d,t) reaction studies. Values of occupation numbers (V_j^2) are determined both by location of levels and by cross-section measurements (the two methods agree), and are used to estimate the unperturbed single-particle states in the shell-model potential. The $g_{7/2}$ state is found to be far above the $d_{5/2}$ (and even above the $s_{1/2}$ and $d_{3/2}$) in contrast to the situation in the tin isotopes and the usual theoretical estimates. The ground state of Zr^{97} is found to be $\frac{1}{2}^+$.

ELASTIC SCATTERING OF POLARIZED 2.8-Mev NEUTRONS. K. V. K. Iyengar and R. A. Peck, Jr.,

Department of Physics, Brown University, Providence, Rhode Island (Received September 18, 1961).

The polarization produced in the elastic scattering at 90° of partially polarized 2.8-Mev neutrons by C, Mg, Al, Ni, and Cu has been measured. All yield positive polarization. The polarizations produced by Ni and Cu are nearly the same whereas those produced by Al and Mg are very different. The polarization observed for carbon is in agreement with calculations from available phase-shift data.

n-p TRIPLE SCATTERING PARAMETERS *R* AND *A*. R. A. Hoffman,* J. Lefrançois,† E. H. Thorndike,‡ and Richard Wilson, Cyclotron Laboratory, Harvard University, Cambridge, Massachusetts (Received September 20, 1961).

The parameters *R* and *A* have been measured for scattering of 140-Mev polarized protons by neutrons bound in deuterium (quasi-free *p-n* scattering). Corrections relating these results to the free *n-p* parameters have been applied, and the following values obtained:

$\theta_{c.m.}$	R_{np}	A_{np}
42.1°	0.228 ±0.119	-0.037 ±0.094
52.5°	0.090 ±0.094	0.080 ±0.074
62.9°	-0.012 ±0.075	0.226 ±0.090
73.4°	-0.147 ±0.095	0.137 ±0.107
83.6	-0.147 ±0.210	0.535 ±0.222

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PROTON-PROTON COLLISIONS AT 4.2 Bev. M. H. Blue,* J. J. Lord, J. G. Parks, and C. H. Tsao, Department of Physics, University of Washington, Seattle, Washington (Received March 27, 1961; revised manuscript received October 5, 1961).

Interactions between 4.15-Bev protons and the free hydrogen nuclei in nuclear emulsion are examined. The total elastic cross section from 27 events was determined to be 11.0 ± 2.6 millibarns. On the basis of 113 interactions the total inelastic cross section was found to be 28.1 ± 3.1 mb. The partial cross sections corresponding to inelastic collisions having two, four, six, and eight secondary particles were found to be, respectively, 16.3 ± 2.4, 11.5 ± 1.8, 0.2 ± 0.1, and 0.1 ± 0.1 mb. While the total inelastic cross section varies slowly with energy, the partial inelastic cross sections were found to be strongly energy dependent. The observed angular distribution of elastically scattered protons in the center-of-mass system was sharply peaked in the forward and backward directions, in fair agreement with calculations based on a simple optical model applicable for energies between 2 and 10 Bev. Particles produced in inelastic collisions were identified as pions or protons by measurements of energy loss and multiple scattering. For those particles identified, center-of-mass system distributions of energy, angle, and transverse momentum are presented.

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FUNDAMENTAL CONSTANTS OF THE MUON. G. Shapiro* and L. M. Lederman, Columbia University, New York, New York (Received September 22, 1961).

Three precise measurements exist on static properties of the muon. These are the *g*-factor, *g*-2, and the frequency of the 3*D*-2*P* transition in mesonic phosphorus. They are combined to obtain the best fit to the fundamental constants of the muon.

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DECAY OF NEUTRAL PIONS. Hla Shwe, Frances M. Smith, and Walter H. Barkas, Lawrence Radiation Laboratory, University of California, Berkeley, California (Received September 5, 1961).

The proper mean life of the neutral pion is measured by a new method which utilizes a large relativistic time dilation. Negative pions of 3.5 Bev were permitted to make interactions in *K*.5 nuclear emulsion. High-energy neutral pions fro some 2000 interaction stars were observed when they decayed by the Dalitz mode. The distances from the star centers at which they decayed were accurately measurable. The momentum spectrum

of the neutral pions was assumed to be the same as that of the charged pions produced in the same interactions. Angular distributions of the secondary charged and neutral pions are presented separately. The mean transverse momentum of the charged pions was measured and found to be 250 ± 10 Mev/c. The neutral pion mean life is $(1.9^{+1.3}_{-0.8}) \times 10^{-16}$ sec.

DEPOLARIZATION IN 212-Mev QUASI-ELASTIC p-n SCATTERING. Robert E. Warner* and John H. Tinlot, Department of Physics and Astronomy, University of Rochester, Rochester, New York (Received September 28, 1961).

The triple scattering parameter D for 212-Mev quasi-elastic p-n scattering from deuterium has been measured at c.m. angles of 40°, 50°, 60°, 70°, and 80°. Protons and recoil neutrons from a liquid deuterium target were detected in coincidence. The experimental results are compared with the theoretical predictions of Breit and coworkers.

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BOSON PHOTOPRODUCTION EXPERIMENTS. Michael J. Moravcsik, Lawrence Radiation Laboratory, University of California, Livermore, California (Received September 13, 1961).

A general discussion is presented of all possible experiments on photoproduction of spin-zero bosons from spin-$\frac{1}{2}$ fermions. Both scalar and pseudoscalar bosons are considered. The M matrix is constructed in terms of four particularly convenient invariants. There are altogether 64 different experiments possible. Of these, 16 are needed to measure all the bilinear combinations of the coefficients of the invariants in the M matrix. To determine the coefficients themselves (not necessarily unambiguously), seven experiments have to be carried out. The 64 observables are given in terms of the bilinear combinations of the coefficients. An extensive list of experiments is then given, each of which determines the parity of the boson participating in the process. All of these experiments require polarized photon sources, and even the simplest one requires the measurement of recoil fermion polarization and the use of a polarized target, although not simultaneously. Sets of seven experiments are then constructed to determine the four coefficients in the M matrix. The simplest of these sets implies the same experimental requirements as the simplest parity experiment. The simplest set of 16 experiments giving all bilinear combinations of the coefficients involves in addition also simultaneously recoil-nucleon polarization measurements and the use of polarized targets.

PION-PION SCATTERING AND LOW-ENERGY PION PRODUCTION. Howard J. Schnitzer,* Department of Physics and Astronomy, University of Rochester, Rochester, New York (Received August 7, 1961).

A model for the reaction $\pi + N \to 2\pi + N$ at low energies with π-π scattering lengths as parameters, discussed in a previous analysis, is extended by removing the assumption that $a_0/a_2 = \frac{5}{2}$. The parameters are determined by using the π^+ angular distribution from $\pi^- + p \to \pi^- + \pi^+ + n$ at 430 Mev and the total cross section for $\pi^+ + p \to \pi^+ + \pi^+ + n$ at 470 Mev. We find two acceptable sets for the π-π scattering lengths: $a_0 = 0.50\,\mu^{-1}$, $a_1 = 0.07\,\mu^{-1}$, $a_2 = 0.16\,\mu^{-1}$, and $a_0 = 0.65\,\mu^{-1}$, $a_1 = 0.07\,\mu^{-1}$, $a_2 = -0.14\,\mu^{-1}$. The total cross sections for the various charge channels and for single partial waves are computed. The predictions of this model are in qualitative agreement with experiment.

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COHERENT PHOTOPRODUCTION OF π^0 FROM DEUTERIUM. Fokion T. Hadjioannou,* Department of Physics, Institute of Theoretical Physics, Stanford University, Stanford, California (Received June 27, 1961).

The calculation of elastic photoproduction of π^0 mesons from deuterium, $\gamma + d \to \pi^0 + d$, is carried out in the impulse approximation at photon energies around 500 Mev. The single nucleon photoproduction amplitudes are taken from dispersion formulas and are corrected for kinematic effects due to internal momentum of the nucleons in the deuteron. We include the D-state part of the deuteron wave function and use different models with Yukawa-type or repulsive core wave functions. We give formulas connecting the cross section with the deuteron form factors. For small momentum transfers the formulas are of course model independent and reduce to the

usual ones. The presence of a 7% D state in a repulsive core model leads to a cross section which falls typically more slowly at high momentum transfers; e.g., at a momentum transfer 2.74 f^{-1}, the cross section is larger by about 40% than the cross section calculated in the absence of the D state. The experimental points favor this model.

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STRANGE PARTICLE PRODUCTION IN PROTON-PROTON COLLISIONS. Tsu Yao, Columbia University, New York, New York (Received September 15, 1961).

An estimate is made for strange particle production in p-p collisions based on the single pion exchange model. For the three-particle final state (KYN), fair agreement with experiment is achieved in both the total cross sections and the momentum and angular distributions of the final particles. For the four-particle final state ($KYN\pi$), general qualitative agreement is achieved for all total cross sections but one. In this, the case of $\pi^0+p+K^+ + \Lambda^0(\Sigma^0)$ production, it is suggested that perhaps Y^* production in p-p collisions plays an important role.

RELATIVISTIC LEE MODEL. Marian Günther,* Institute for Advanced Study, Princeton, New Jersey (Received August 7, 1961).

A relativistic version of the well-known exactly soluble Lee model of quantum field theory is constructed, which—unlike the original nonrelativistic Lee model—fulfills also the condition of microcausality (field operators commuting or anticommuting for space-like distances). Microcausality is, however, found compatible with solubility only if an indefinite metric is introduced on a much larger scale than was necessary in the nonrelativistic case.

Notwithstanding the fact that a formalism of this type would seem to bear still less resemblance to physical reality than does the original Lee model, we arrive at a theory almost identical with the usual version of relativistic charged scalar (or pseudoscalar) meson field theory. From a certain point of view these two formalisms can even be considered identical. In fact, the difference between the realistic theory and the present Lee model hinges merely on whether one uses, for the representation of the Dirac standard ket $|0\rangle$ a physical vacuum or a suitably defined "completely empty space."

The form of the presentation follows closely that of the well-known Pauli-Källén description of the nonrelativistic Lee model, up to the point where it is shown that some of the main results can also be obtained with help of a corresponding Chew-Low equation.

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QUASI-CLASSICAL THEORY OF THE NONSPINNING ELECTRON. Ralph Schiller,* Syracuse University, Syracuse, New York (Received August 31, 1961).

We develop a modified Hamilton-Jacobi theory of classical mechanics following the early work of Van Vleck. This modified Hamilton-Jacobi theory, or quasi-classical theory, permits us to exhibit in classical mechanics many features that in the past have been exclusively associated with quantum mechanics. We deal with classical wave functions, classical operators, classical "eigenvalue" equations, a classical "sum over paths" formulation of classical mechanics, and with classical creation and destruction operators. Following Van Vleck, one can derive the WKB approximate solutions to the Schrödinger equation from the solutions of the classical Hamilton-Jacobi equation. If we apply the methods of Keller to the nonrelativistic and relativistic Kepler problem, we derive eigenvalues from the requirement of single-valuedness imposed on the WKB solutions. It turns out that the energy eigenvalues are those given by the Schrödinger equation and the Klein-Gordon equation, respectively. In the particular case of the harmonic oscillator there exists a canonical transformation which transforms the quasi-classical equation into an exact equation of quantum mechanics. We conjecture that if the WKB approximation and the Schrödinger equation predict the same eigenvalues, then there always exists a canonical transformation which transforms the quasi-classical equation into the corresponding Schrödinger equation. Finally we derive the quasi-classical equations in momentum space.

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QUASI-CLASSICAL TRANSFORMATION THEORY. Ralph Schiller,* Syracuse University, Syracuse, New York (Received August 31, 1961).

The quasi-classical theory is the theory of the WKB approximation as defined by Van Vleck. This approximate wave theory is invariant under canonical transformations. We derive the transformation laws for wave functions and operators in this theory. The connection problem of the ordinary configuration space is discussed from the point of view of Van Vleck. The usual method of WKB quantization is then contrasted with another method of quantization in a classical configuration space of creation and destruction operators. In this new configuration space we find that, for some problems, the quasi-classical solutions are exact solutions of the Schrödinger equation. For this class of problems one may employ purely classical techniques to find solutions of the Schrödinger equation. Finally we show that the canonical transformations of the quasi-classical theory may be put into one-to-one correspondence with a group of approximate unitary transformations.

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QUASI-CLASSICAL THEORY OF THE SPINNING ELECTRON. Ralph Schiller, Stevens Institute of Technology, Hoboken, New Jersey (Received September 1, 1961).

We derive a modified Hamilton-Jacobi theory for a classical spinning dipole and show that this classical theory can be put in a form almost identical with the Pauli spin theory. We quantize this theory by requiring that a classical spinor "wave function" be continuous and single-valued, and that it satisfy the usual energy eigenvalue equation. In this manner we deduce the correct energy levels for a hydrogen atom in an external magnetic field. We derive the integral kernel for the Pauli equation from this classical model and discuss the properties of our spinor solutions under canonical transformation. We exhibit the charge-conjugate solution to the classical spin equation.

RENORMALIZATION OF MANY-FERMION MOMENTUM-SPACE DISTRIBUTIONS IN HIGHER RANDOM PHASE APPROXIMATIONS. N. R. Werthamer and H. Suhl, University of California, San Diego, La Jolla, California (Received September 5, 1961).

The approach to the many-fermion problem known as the method of higher random phase approximations (RPA) is given a more rigorous formulation. It is shown that the previous heuristic procedures for evaluation of the second RPA are justified, in that expectation values of plane wave operators with respect to the true ground state may validly be replaced by their values in the unperturbed Fermi state. This property of momentum-space occupation renormalization is conjectured to hold also to higher orders of RPA than the second.

CHIRALITY CONSERVATION AND SOFT PION PRODUCTION. Y. Nambu and D. Lurié, The Enrico Fermi Institute for Nuclear Studies and Department of Physics, University of Chicago, Chicago, Illinois (Received September 25, 1961).

A formally γ_5-invariant system consisting of a Dirac field and a massless pseudoscalar field allows chirality conservation in the sense that its expectation value is a constant of motion. This leads to the consequence that in any reaction a change in the fermion chirality (~helicity×velocity) is compensated for by the emission of a massless boson at zero energy, which can be expressed by a simple formula relating the radiative amplitude to the elastic amplitude.

Assuming the pion-nucleon system to be γ_5-invariant when the pion mass can be neglected, the formula is applied to the processes $N+\pi \to N+\pi$ and $N+2\pi$. A reasonable agreement with experiment is obtained in a case dominated by the 3-3 resonance.

RUNAWAY MODES IN MODEL FIELD THEORIES. Sidney Coleman, California Institute of Technology, Pasadena, California, and Richard E. Norton, University of California, Los Angeles, California (Received September 27, 1961).

Within the framework of linear quantum field theories, a general study is presented of the existence and removal of runaway modes—solutions of the equations of motion which exhibit a real exponential time dependence. It is hoped that this work will yield insight into the corresponding problem in physically realistic theories. It is

shown that runaway modes occur only when the Hamiltonian is not positive definite, and that they occur in linear quantum theories whenever they appear in the corresponding classical theories. Three methods are proposed for eliminating these unphysical modes. One is the analog of the method used by Dirac in classical electron theory; the other two are believed to be new.

ON THE UNIQUENESS OF FOCK'S HARMONIC COORDINATE SYSTEMS IN THE PRESENCE OF STATIC, SPHERICALLY SYMMETRIC SOURCES. F. J. Belinfante and J. C. Garrison,* Department of Physics, Purdue University, Lafayette, Indiana (Received September 13, 1961).

Fock has claimed that his "harmonic" coordinate systems in curved space flattening out toward spatial infinity are uniquely determined but for an arbitrary inhomogeneous Lorentz transformation. If this is so, introduction of Fock's harmonic coordinate conditions would provide a natural way of introducing a Lorentz subgroup of the general coordinate transformation group of Einstein's gravitational theory, and of defining a Minkowski metric besides the curved-space metric. This would open the way to close relations between Einstein's gravitational theory on the one hand, and Lorentz-covariant quantum field theory on the other hand.

A general proof of the correctness of Fock's claim, for universes satisfying his boundary conditions, has never been given rigorously. Here we extend an earlier proof of this uniqueness for the Schwarzschild field around a single gravitational singularity, to the case of the static and spherically symmetric field generated in some coordinate system by an extended static and spherical distribution of energy and of stresses. The uniqueness (but for the zero point of time and for a spatial rotation) of the harmonic coordinate system in which this field is spherical and at rest around the spatial origin is here guaranteed by the condition that there must be a one-to-one correspondence between the points x, y, z, t of the harmonic coordinate system and the points in physical space.

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CAN MASSLESS PARTICLES BE CHARGED? K. M. Case* and S. G. Gasiorowicz,† Lawrence Radiation Laboratory, University of California, Berkeley, California (Received September 25, 1961).

It is shown that for particles of spin one or larger, Lorentz invariance, masslessness, and conventional electromagnetic coupling are mutually incompatible.

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SPECTRAL MOMENTS AND CONTINUUM PERTURBATION THEORY. E. O. Kane, Bell Telephone Laboratories, Murray Hill, New Jersey (Received August 31, 1961).

The pth spectral moment is shown to be given exactly by the perturbation expansion of the resolvent operator carried to order p. Any individual term of order n in the perturbation expansion makes an identically zero contribution to the pth moment for n greater than p. The above is true even though perturbation theory is nonconvergent. The result may be useful in cases where perturbation theory converges for high energy but not for low energy. An application to the problem of band structure "tails" in impure crystals is suggested.

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ELECTROMAGNETIC RADIATION FROM SOLAR CORPUSCULAR STREAMS

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(Received August 29, 1961)

Cyclotron radiation from solar corpuscular streams (SCS) moving in the earth's plasma is considered in MacArthur's pioneer paper.[1] But he considers only the waves with wave vector $\vec{k}$ parallel to the earth's magnetic field H ($\vec{k} \parallel \vec{H} \parallel OZ$); moreover he confines himself only to sublight motion (by which we mean motion with velocity u less than the phase velocity $v_f = \omega/k$ of emitted waves). Our note presents some objections to his paper[1] and also studies radiation for superlight motion (when $u > v_f$). We confine ourselves to the linear approximation in n'/n, where n and n' are, respectively, the plasma and stream density.

Since the ion motion is sublight ($u < v_f$), it does not emit any cyclotron radiation with $\vec{k} \parallel \vec{H}$. This follows[2] from the expressions for radiation intensity obtained by Eidman.[3] So the formulas[1] should be generalized for oblique waves, i.e., for waves with $\theta \neq 0$, where θ is the angle between $\vec{k}$ and $\vec{H}$; $OY \parallel \vec{k} \times \vec{H}$.

The motion of SCS in a broad range of frequencies is a superlight one.[4] Therefore the amplitude of coherent oscillations excited by the stream increases with time [correspondingly, in space an amplification takes place: $E, h \sim e^{\mu \cdot \vec{r}} e^{i(\vec{k} \cdot \vec{r} - \omega t)}$, a cyclotron instability[2,4-6]], and it may exceed the amplitude of incoherent radiation considered by MacArthur.

The superlight character of the motion changes also the frequency spectrum. When $u < v_f$, an ion radiates only at frequencies $\omega > \Omega$ ($\Omega = eH/Mc$ = gyromagnetic frequency of plasma ions); when $u > v_f$, it radiates also at frequencies $\omega < \Omega$ (cyclotron and Čerenkov excitation of hydromagnetic waves).

Radiation frequencies at the onset of instability are found from the equation of the superlight Doppler effect:

$$\frac{u}{v_A} = \left(1 + \frac{\gamma Q}{\eta}(1-\beta^2)^{1/2}\right)\frac{N_A}{N\cos\theta}, \qquad (1)$$

where $\eta = \omega/\Omega$; $\beta = u/c$; $\omega_H = eH/mc$ and $\Omega' = eH/M'c$ = electron and stream-ion gyromagnetic frequencies, respectively; $N(\theta, \eta)$ = plasma refractive index[7]; $\gamma = 0, 1, 2, \cdots$ is the order of the harmonics ($\gamma = 0$ corresponds to Čerenkov excitation[8,9]); $v_A = H(4\pi nM)^{-1/2}$ = the Alfvén velocity; $Q = \Omega'/\Omega$; $N_A = c/v_A$.

Figure 1 shows the solutions of (1): η is plotted against u/v_A. Let us denote the Alfvén (ordinary) wave by A wave, the magnetoacoustical (extraordinary) wave by B wave. In Fig. 1 cyclotron excitation is represented by the curves 1-2 (B wave, $\theta = 0°, 45°$), 3-4 (A wave, $\theta = 30°, 45°$), and 8 (MacArthur's formula for $u < v_f$, $\theta = 0$); Čerenkov excitation by the curves 5-6 (A wave, $\theta = 30°, 45°$) and 7 (B wave, $\theta = 45°$).

For $\omega < (\omega_H \Omega)^{1/2}$, (1) gives

$$\frac{u}{v_A} = \left(1 + \frac{\gamma Q}{\eta}\right)\left\{\frac{2(1-\eta^2)}{1 + \cos^2\theta \pm \left[\sin^4\theta + (4\cos^2\theta)\eta^2\right]^{1/2}}\right\}^{1/2}. \qquad (2)$$

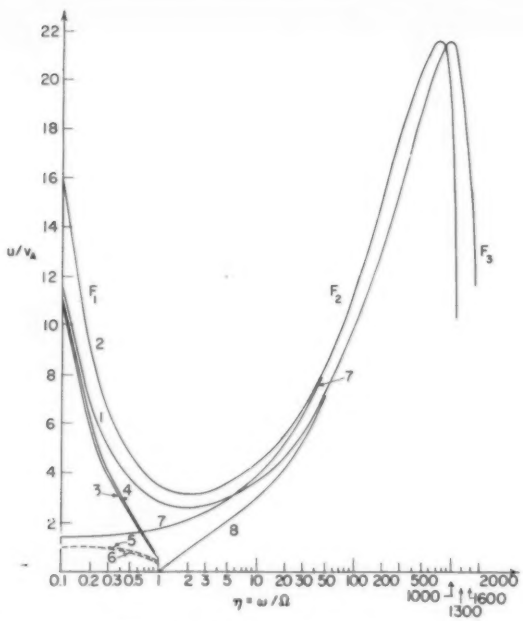

FIG. 1. Solutions of Eq. (1).

(The plus sign corresponds to A wave, minus to B wave.)

For $\omega \ll \Omega$,

$$\eta = rv_A/u \ (A \text{ wave}); \quad \eta = rv_A/u\cos\theta \ (B \text{ wave}). \quad (3)$$

For $\omega > (\omega_H\Omega)^{1/2}$ (whistlers),

$$\frac{u}{v_A} = \left(1 + \frac{Qr}{\eta}\right)\frac{[\eta(\cos\theta - \alpha\eta)]^{1/2}}{\cos\theta}. \quad (\alpha = m/M = 1/1836) \quad (4)$$

The frequencies at $u < v_f$ and $u > v_f$ are almost equal; however, the processes are different (cyclotron instability appearing only at $u > v_f$).

Our result on simultaneous excitation in three frequency ranges (F_1, F_2, F_3—Fig. 1) is confirmed by observations.[10] The other conclusion—that the frequency increases as the stream speed decreases—fits the observations.[11] (Stream deceleration increases[3] the micropulsation frequency F_1 and diminishes the frequency F_2.)

The amplification factor μ may be determined from the equation $(k + i\mu)^2 = (\omega^2/c^2)\bar{N}^2$, where $N^2(\epsilon_{\alpha\beta} + \epsilon_{\alpha\beta}', \theta)$ = index of refraction of the system, plasma + SCS; $\epsilon_{\alpha\beta}, \epsilon_{\alpha\beta}'$ = components of the plasma and stream ($\epsilon_{\alpha\beta}'$) dielectric constants, respectively, computed by Stepanov and Kitsenko[5] and by Gintsburg.[12] Let S be the thermal velocity of SCS ions [$S = (2kT')^{1/2}M'^{-1/2}$] and $\lambda = (S/u)(1+\eta)\tan\theta = (k\sin\theta)S/\Omega$. Then (provided $n' \ll n$)[13]

$$\mu = -\frac{\omega}{c}\left(\frac{\partial F}{\partial N}\right)^{-1}\sum_{\alpha,\beta=1}^{3}\left[\frac{\partial F}{\partial \epsilon_{\alpha\beta}}\right]\epsilon_{\alpha\beta}', \quad (5)$$

where

$$F = |N^2\delta_{ij} - N_iN_j - (\epsilon_{ij} + \epsilon_{ij}')|,$$

in which $N_1 = N\sin\theta$, $N_2 = 0$, $N_3 = N\cos\theta$; N is the refractive index of the plasma, which is obtained by setting $\epsilon_{ij}' = 0$ and solving $F = 0$.

When $\omega \ll \Omega$, $\mu_A^{\text{cycl}} = -\frac{1}{2}(\Omega/\omega)(\epsilon_{xx}'/\epsilon_0)$ (A wave) $\mu_B^{\text{cycl}} = -\frac{1}{2}(\Omega/u)(\epsilon_{yy}'/\epsilon_0)$ (B wave), where $\epsilon_0 = N_A$. Inserting the expressions for ϵ_{xx}' and ϵ_{yy}',[5,12] we have (for cyclotron excitation):

$$\mu_A^{\text{cycl}} = \pi^{1/2}\frac{n'}{n}\frac{u^2}{v_A^2}\frac{\Omega}{S\cos\theta}r^2\frac{\exp(-\lambda^2/2)}{\lambda^2}I_r(\lambda^2/2);$$

$$\mu_B^{\text{cycl}} = \mu_A^{\text{cycl}}\cos^2\theta. \quad (6)$$

At $\lambda \ll 1$, Eq. (6) gives

$$\pi\frac{e}{c}\frac{n'u^2}{HS\cos\theta} \ (e\text{-ion charge}).$$

For Čerenkov excitation ($\eta \ll 1$, $\lambda \ll 1$):

$$\mu_A^{C} = -\tfrac{1}{2}(\epsilon_{yy}'/\epsilon_0)\eta^2 k\cot^4\theta = \tfrac{1}{2}(\pi/2)^{1/2}\frac{n'}{n}\frac{S^2}{v_A^2}\eta^2 k\cot^2\theta$$

$$\mu_B^{C} = \tfrac{1}{2}(\pi/2)^{1/2}k(S^2/v_A)\sin^2\theta.$$

So

$$\mu^{\text{cycl}}/\mu^{C} > (u^2/S^2)(v_A/S) \gg 1,$$

so that the cyclotron mechanism prevails over the Čerenkov mechanism.

Cyclotron and Čerenkov radiations are emitted also by relativistic particles. From reference 1 we find the frequency of hydromagnetic waves radiated by inner radiation belt protons:

$$\omega = \Omega(v_A/c)QR^{-1}, \quad R = \left[\frac{W}{M'c^2}\left(\frac{W}{M'c^2} + 2\right)\right]^{1/2},$$

where W = proton kinetic energy. At $W = 75$ to 750 Mev the frequencies of such micropulsations ≈ 0.2 to 1.5 cps. Relativistic electrons of the outer radiation belt give cyclotron radiation in the range

2 to 4 kc/sec [$\omega = \omega_H \cos\theta(1+\alpha R^2 N_A{}^2 \cos^2\theta)^{-1}$].

The SCS radiate A waves in the band $\omega < (v_A/u)\Omega \ll \Omega$, and B waves in the band $(v_A/u)\Omega < \omega \ll \Omega$ by the cyclotron mechanism. (See Fig. 1.) But at $\Omega \gg \omega$ the group velocity of A waves is directed, regardless of the orientation of the wave vector $\vec{k}$, along the magnetic field H; so at the earth's surface the wave will have the polarization and frequency of an A wave (neglecting the possibility of wave-type transformation, due to inhomogeneities of the medium or of the field). Such polarization was observed by Sugiura.[14]

However, micropulsations[14] might be explained by one more type of instability (considered for electron streams by Zhelezniakov[15])—transverse impulse instability.[16]

When $\omega > \Omega$ the B wave only remains, so micropulsations and radio emission should have elliptical polarization of opposite sense.

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[16]This instability can also occur for sublight motion.

STABILITY CRITERION FOR ARBITRARY HYDROMAGNETIC EQUILIBRIA*

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(Received November 13, 1961)

A condition which is both necessary and sufficient for the stability of general hydromagnetic equilibria with respect to localized displacements has been derived. Such localized instabilities have been studied for many special configurations,[1-5] and are often severely limiting.[6]

In a hydromagnetic fluid model, an equilibrium configuration is determined by the equations

$$\vec{J} \times \vec{B} = \vec{\nabla} p, \tag{1}$$

$$\vec{J} = \vec{\nabla} \times \vec{B}, \tag{2}$$

$$\vec{\nabla} \cdot \vec{B} = 0. \tag{3}$$

The general solutions of these equations for systems which are spatially bounded have the property that the lines of force lie on surfaces, called magnetic surfaces, which are topologically nested toroids.[7] On some surfaces the magnetic lines of force may close on themselves after n revolutions the short way around the toroid and m revolutions the long way. We define the rotational transform for such a surface by

$$\iota = 2\pi n/m. \tag{4}$$

It is the same for all lines of force on the surface. The definition of the rotational transform can be extended so that ι is continuous. We assume that the rotational transform varies from surface to surface. Then both the set of surfaces for which $\iota/2\pi$ is rational and the lines close on themselves, and the set for which $\iota/2\pi$ is irrational and the lines are ergodic on the surface, are dense sets. We also assume that $\vec{\nabla} p$ does not vanish.

The stability problem is tractable only in a suitably chosen coordinate system. We adopt a nonorthogonal coordinate system in which a magnetic surface is labeled by the volume V which it contains and a line of constant θ and ζ intersects each surface once. Hamada[8] has shown that θ and ζ can be

chosen such that

$$\vec{\nabla}V\cdot\vec{\nabla}\theta\times\vec{\nabla}\zeta=1, \tag{5}$$

for a system in which the magnetic field is given by

$$\vec{B}=\psi'(V)\vec{\nabla}V\times\vec{\nabla}\theta+\chi'(V)\vec{\nabla}\zeta\times\vec{\nabla}V. \tag{6}$$

Here ψ is the flux through a constant ζ cross section of the surface, χ is the flux through a constant θ ribbon, one side of which lies in the surface and the other on the magnetic axis ($V=0$), and primes denote derivatives with respect to V.

We use the energy principle of Bernstein et al.,[9] which states that the system is stable if and only if the functional

$$2\delta W=\int\left\{\left[\vec{\nabla}\times(\vec{\xi}\times\vec{B})+\frac{\vec{J}\times\vec{\nabla}V}{|\vec{\nabla}V|^2}\vec{\xi}\cdot\vec{\nabla}V\right]^2+\gamma p(\vec{\nabla}\cdot\vec{\xi})^2+K(\vec{\xi})^2\right\}dV\,d\theta\,d\zeta, \tag{7}$$

where

$$K=-2[\vec{J}\times\vec{\nabla}V\cdot(\vec{B}\cdot\vec{\nabla})\vec{\nabla}V]/|\vec{\nabla}V|^4, \tag{8}$$

can not be made negative for some $\vec{\xi}(V,\theta,\zeta)$. For localized displacements this can be reduced by a series of algebraic minimizations to a functional of $\vec{\xi}\cdot\vec{\nabla}V$. From this form it can be shown that the necessary and sufficient condition for stability with respect to localized displacements is that

$$F=\left\{\left[\tfrac{1}{2}(\psi'\chi''-\psi''\chi')\oint\frac{dl}{|\vec{B}|}-\oint\frac{\vec{J}\cdot\vec{B}}{|\vec{\nabla}V|^2}\frac{dl}{|\vec{B}|}\right]^2\Big/\oint\frac{|\vec{B}|^2}{|\vec{\nabla}V|^2}\frac{dl}{|\vec{B}|}\right\}+\oint K\frac{dl}{|\vec{B}|}>0, \tag{9}$$

on each closed line of force, where the integrals are evaluated along that line of force.

Mercier[10] has given a necessary condition for the stability of these equilibria with respect to localized perturbations which corresponds to Eq. (9) but with the line integrals replaced by integrals over the magnetic surfaces. For helically invariant systems, which represent the most general two-dimensional configurations, each line integral is the same for all lines of force on the surface and Eq. (9) reduces to Mercier's condition.

Details of the calculation will be forthcoming.

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LOW ELECTRON EFFECTIVE MASSES AND ENERGY GAP IN $Cd_xHg_{1-x}Te$

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Electron effective masses as low as $0.003m_0$ and an energy gap of 0.025 ev have been measured in plasma and interband magnetoreflectivity experiments on alloys of the composition Cd_x-$Hg_{1-x}Te$, where x is in the range 0.1-0.2. Hall mobilities as high as 5.4×10^5 and 1.1×10^6 cm^2/v-sec have been observed in these materials at 77°K and 4.2°K, respectively. Lawson et al.[1] have previously reported a Hall mobility of 2.7×10^5 cm^2/v-sec at 77°K for $Cd_{0.1}Hg_{0.9}Te$.

Undoped ingots are prepared from the high-purity elements by the Bridgman technique. The samples are n-type over the entire temperature range investigated, 4.2°-300°K. Values of carrier concentration ($n=1/R_He$) and Hall mobility ($R_H\sigma$) at three temperatures are listed in Table I for a typical sample (No. 8B). The carrier concentration and Hall mobility do not depend strongly on alloy composition over the limited range investigated, as illustrated by the data for 4.2°K listed in Table II. The measurements of electron effective mass described below indicate that the extraordinarily high electron mobilities at the lower temperatures are due to very low effective masses rather than to unusually long relaxation times. Since the data are quite limited, the systematic decrease in carrier concentration at 4.2°K with increasing Cd content may be fortuitous.

The dependence of transverse magnetoresistance ($\Delta\rho/\rho_0$) and Hall coefficient (R_H) on magnetic field has been measured between 3 and 12 kgauss for one sample at 4.2°K. The value of $\Delta\rho/\rho_0$ increases strongly with increasing field, reaching approximately 30 at 12 kgauss. The value of R_H decreases by about 5% from 3 to 12 kgauss. Neither property changes smoothly with field. Although the observed irregularities are insufficient in number and magnitude to permit detailed analysis, we believe that they are quantum effects which result from the magnetic splitting of the conduction band, like the oscillatory magnetoresistance observed in InSb[2] and InAs.[3,4] The fact that R_H does not increase with field indicates that sample inhomogeneities do not lead to erroneously high values of R_H and $R_H\sigma$, as they do for some samples of InSb.[5] The fact that R_H exhibits only a small decrease with increasing field indicates that holes do not carry an appreciable fraction of the current and therefore that use of the one-carrier expression $n=1/R_He$ is justified.

Optical transmission through a specimen 40 μ thick prepared from sample No. 8B has been measured at room temperature for wavelengths between 2 and 25 μ. The relative transmission first became measurable at 10 μ, increased to a maximum of 28% at 20 μ, and then gradually decreased. Interference fringes were observed beginning at 15 μ. A value of 4.1 for the refractive index is calculated from the spacing of the fringes. The position of the diffuse absorption edge at short wavelengths, which is in reasonable agreement with Lawson's results for absorption edge versus alloy composition,[1] indicates that interband transitions can be produced by photons with energies as low as 0.12 ev. The decrease in transmission beginning at 20 μ is presumably due to free-carrier absorption.

Preliminary measurements of the plasma edge due to free electrons have been carried out on sample No. 8B in the far infrared with an $F/2$ spectrometer. The plasma edge and its shift with temperature and magnetic field are shown in Fig. 1. From the curves at $H=0$, we obtain an average dielectric constant $K=15$, in reason-

Table I. Characteristics of $Cd_{0.14}Hg_{0.86}Te$ (Sample No. 8B).

T (°K)	$n\times10^{-16}$ (cm^{-3})	$R_H\sigma\times10^{-4}$ (cm^2/v-sec)	m^*/m_0
300	13	2.7	0.015
77	1.5	40	0.005
40	0.6	56	0.003

Table II. Characteristics of $Cd_xHg_{1-x}Te$ at 4.2°K.

Sample	x	$n\times10^{-15}$ (cm^{-3})	$R_H\sigma\times10^{-5}$ (cm^2/v-sec)
8B	0.136	3.2	6.2
8LA	0.176	2.3	6.6
7-1[a]	0.196	1.9	11
11LA	0.20	1.7	5.9

[a]Single crystal.

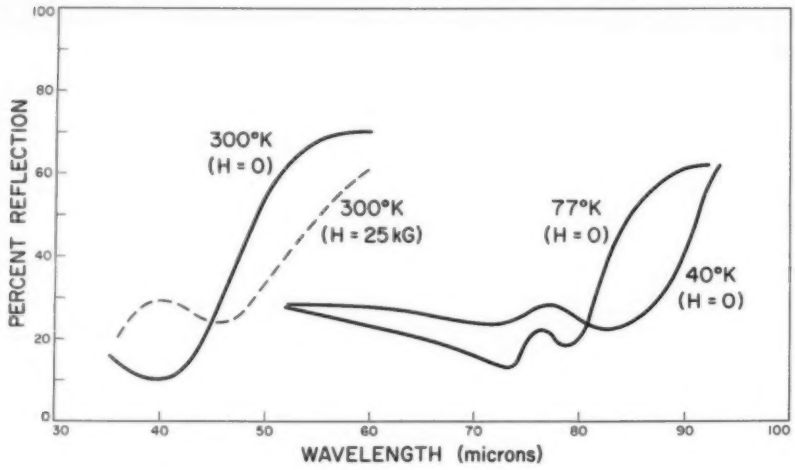

FIG. 1. Plasma edge for $Cd_{0.14}Hg_{0.86}Te$ (Sample No. 8B).

able agreement with the value of 16.8 calculated from the refractive index. The effective masses calculated from the data are shown in Table I. Since the mass varies with concentration and therefore with Fermi level, the conduction band is nonparabolic. The position of the reflection maximum observed at 77 μ is independent of temperature and magnetic field. This suggests that this maximum is associated with the lattice rather than the electrons and is probably a reststrahlen peak.

We have observed interband transitions by magnetoreflection measurements[6,7] on sample No. 8B at 77°K. The results are shown in Fig. 2, where the photon energy required for an interband transition is plotted against magnetic field. Each line represents a different transition. By extrapolation of the lines to zero magnetic field we obtain an energy gap of 0.025 ev. Although a precise value of effective mass cannot be obtained from these data, the estimated value is consistent with the one obtained from the plasma measurements at 77°K.

The energy gap obtained from the magnetore-

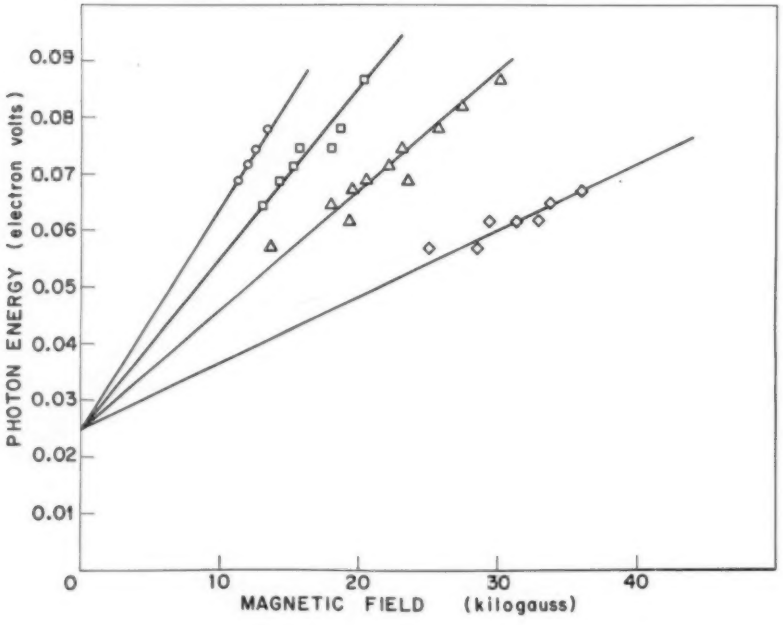

FIG. 2. Interband magnetoreflection for $Cd_{0.14}Hg_{0.86}Te$ (Sample No. 8B) at 77°K.

flection experiments is much less than the absorption edge energy of 0.12 ev. The two results are not inconsistent, however, since the absorption edge energy is approximately the sum of an energy gap and the Fermi level, ϵ_F.[8] Although our effective-mass data are too limited to permit an accurate calculation of ϵ_F, we obtain a rough estimate of 0.085 ev for ϵ_F by assuming an average value of $m^* = 0.01\, m_0$. The sum of ϵ_F and energy gap is thus about 0.11 ev, in satisfactory agreement with the absorption edge energy.

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EFFECT OF HYDROSTATIC COMPRESSION ON THE ENERGY OF THE 14.4-kev GAMMA RAY FROM Fe^{57} IN IRON*

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The energy of the recoil-free fraction of the γ rays emitted by nuclei bound in solids[1] has been found to be affected by temperature[2,3] and by electronic configuration.[4-7] The latter effect has been named the "isomeric" shift. Compression of a solid should influence the energy through both of these mechanisms. We have measured the effect of hydrostatic compression at 295°K on the energy $h\nu$ of the recoil-free 14.4-kev γ rays emitted by 0.1-μsec Fe^{57} bound in metallic iron. The temperature coefficient of the energy has also been measured more accurately than before.

The apparatus, described in the block diagram of Fig. 1, was that developed for measurement of the effect of gravity,[8] except that an aluminum-windowed proportional counter 60 cm deep having an argon-methane mixture at atmospheric pressure was used. This counter made possible a counting rate up to 2×10^4 per second in the pulse-height channel corresponding to 14.4 kev by virtue

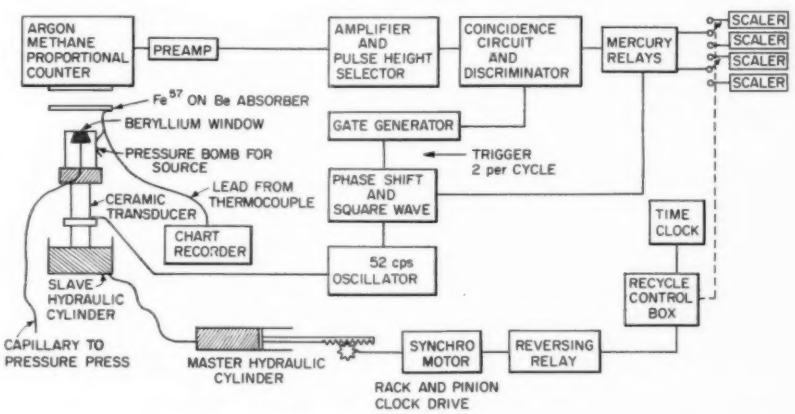

FIG. 1. A block diagram of the apparatus. Alternate five-minute periods automatically record data on separate pairs of scalers, corresponding to upward and downward motion of the hydraulic cylinder. Backlash in the drive is removed during a 30-sec dead time between counting periods.

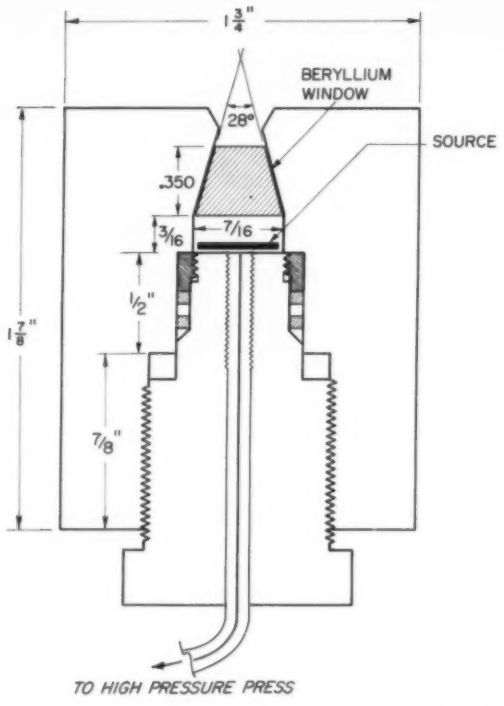

FIG. 2. A cross-sectional drawing of the pressure bomb.

of its small response to the copious initial 120-kev γ ray. The source, about 2 mC of Co^{57} diffused into iron, was enclosed in a beryllium-copper pressure bomb equipped with a one-half-inch thick Be window, as shown in Fig. 2. The bomb was cemented to the ferroelectric transducer and connected to a Bridgman press by a stainless steel tube.

Pressures of one atmosphere, about 2000 kg/cm^2, and 3000 kg/cm^2 were used to determine the pressure coefficient of frequency. The result found was

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial P}\right)_T = -(2.61\pm0.10)\times10^{-13} \text{ per kg/cm}^2. \quad (1)$$

The error assigned is double the standard deviation associated with counting statistics to allow liberally for systematic instabilities. It was not convenient to use a "monitor" channel as in the gravitational experiment. A systematic effect attributable to pressure in the bomb and feed tube, which might be thought to change the transducer waveform, was shown to be absent in separate runs that utilized a source mounted on the outside of the bomb.

The temperature coefficient was measured by comparing the apparent shift between source and absorber when they were at nearly equal temperatures with that found when the absorber (enriched Fe^{57} plated on Be backing) was held about 5°C warmer than the source. As in the gravitational experiment, only differences attributable to the parameter changed were significant owing to the presence of shifts of comparable scale inherent in all source-absorber combinations. In both experiments temperature differences were monitored continuously by the recording thermocouple and variations were taken into account. The result found for the temperature coefficient at 297°K and one atmosphere was

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial T}\right)_P = -(2.09\pm0.05)\times10^{-15} \text{ per °K}, \quad (2)$$

identical to, but more precise than, the earlier measurement.[2]

We suppose that the temperature coefficient may be split into two parts;

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial P}\right)_T = \frac{1}{\nu}\left(\frac{\partial\nu_{\text{isom}}}{\partial \ln V}\right)_T\left(\frac{\partial \ln V}{\partial P}\right)_T + \frac{1}{\nu}\left(\frac{\partial\nu_{\text{rel}}}{\partial \ln V}\right)_T\left(\frac{\partial \ln V}{\partial P}\right)_T. \quad (3)$$

The first term represents the effect of changing the electronic density at the nucleus $|\psi(0)|^2$ and the second the relativistic effect of changing the mean vibrational energy. An estimate of the second contribution may be made from the Debye model which gives, as an approximation, for the vibrational energy;

$$E_{\text{vib}} = 3kT\left[1+\frac{1}{20}\left(\frac{\theta}{T}\right)^2+\cdots\right]. \quad (4)$$

For the Debye temperature θ not much larger than the temperature T, the approximation above is adequate without more terms than written. From this is obtained

$$\frac{1}{\nu}\left(\frac{\partial\nu_{\text{rel}}}{\partial \ln V}\right)_T\left(\frac{\partial \ln V}{\partial P}\right)_T \approx -\frac{3}{20}\frac{k\theta}{Mc^2}\left(\frac{\theta}{T}\right)\left(\frac{\partial \ln\theta}{\partial \ln V}\right)_T\left(\frac{\partial \ln V}{\partial P}\right)_T, \quad (5)$$

where M is the atomic mass of Fe^{57} and Grüneisen's constant γ may be used for $-(\partial \ln\theta/\partial \ln V)_T$. For $\theta\approx400$°K, $\gamma=1.6$,[9] and $(\partial \ln V/\partial P)_T = -5.98\times10^{-7}$ per kg/cm^2,[10] one obtains at $T=295$°K a value for Eq. (5) of approximately -0.126×10^{-13} per kg/cm^2. This is less than 5% of the pressure coefficient

observed. The volume dependence of the isomeric shift must therefore account for the major part of the pressure coefficient. (A related calculation of the effect of pressure on the lattice vibrations and therefore on the amount of the recoil-free fraction has been given by Hanks.[11]) One may also conclude that the net pressure coefficient should vary only weakly with temperature. At $T = 0°K$ the vibrational contribution would be about -0.35×10^{-18} per kg/cm^2. For $T \gg \theta$ this contribution approaches zero. This explicit temperature variation is reduced by the decrease of $(\partial \ln V/\partial P)$ with temperature.

If the pressure coefficient originates mainly in the isomeric shift, one may write

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial P}\right)_T \cong \frac{1}{\nu}\left(\frac{\partial\nu_{\text{isom}}}{\partial \ln V}\right)_T \left(\frac{\partial \ln V}{\partial P}\right)_T = K\left(\frac{\partial|\psi(0)|^2}{\partial \ln V}\right)_T \left(\frac{\partial \ln V}{\partial P}\right)_T . \tag{6}$$

The fractional frequency shift per unit change of electronic density K can be estimated from isomeric shifts measured for iron ions of differing configuration provided that differences of the values of $|\psi(0)|^2$ appropriate to those ions can be estimated. This estimation can be made[7] using calculated free-ion wave functions.[12] This procedure gives

$$K \approx -(1.6\pm0.2)\times10^{-12} \text{ per } a_0^3, \tag{7}$$

where a_0 is the Bohr radius. Equations (6) and (7) lead to

$$\partial|\psi(0)|^2/\partial \ln V = -(2.7\pm0.4)a_0^{-3}. \tag{8}$$

If one were to estimate $\partial|\psi(0)|^2/\partial \ln V$ from the assumptions (1) that only $|\psi_{4S}(0)|^2$ is changed by compressions, and (2) that the $|\psi_{4S}(r)|^2$ scales with the volume, i.e., $(\partial \ln|\psi_{4S}(0)|^2/\partial \ln V) = -1$, then the estimate[7] $|\psi_{4S}(0)|^2 \approx 3a_0^{-3}$ gives $(\partial|\psi(0)|^2/\partial \ln V)_{\text{estimated}} = -3a_0^{-3}$.

The temperature coefficient of γ-ray energy at constant pressure has two contributions:

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial T}\right)_P = \frac{1}{\nu}\left(\frac{\partial\nu}{\partial T}\right)_V + \frac{1}{\nu}\left(\frac{\partial\nu}{\partial P}\right)_T \left(\frac{\partial P}{\partial \ln V}\right)_T \left(\frac{\partial \ln V}{\partial T}\right)_P . \tag{9}$$

The first term is the explicit temperature dependence at constant volume, and the second is the effect of thermal expansion. At 295°K the second term is

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial \ln V}\right)_T \left(\frac{\partial \ln V}{\partial T}\right)_P = +(0.15\pm0.01)\times10^{-15} \text{ per } °K, \tag{10}$$

which is opposite in sign to the observed effect of temperature. This, combined with the remeasured temperature coefficient, Eq. (2), gives

$$\frac{1}{\nu}\left(\frac{\partial\nu}{\partial T}\right)_V = -(2.24\pm0.06)\times10^{-15} \text{ per } °K. \tag{11}$$

It is this quantity which should be compared to the effect derivable as a consequence of time dilatation,[2]

$$\left[\frac{1}{\nu}\left(\frac{\partial\nu}{\partial T}\right)_V\right]_{\text{relativistic}} = -C_{LV}/2Mc^2, \tag{12}$$

where C_{LV} is the specific heat at constant volume of only the lattice and M is the atomic mass. Bijl[13] has extracted values for $C_{LV}(T)$ from specific heat data, and gives a value for iron amounting to 0.935 of the classical value at 295°K. He has not corrected for possible contributions to the measured specific heat from spontaneous magnetization. Using the above value, one finds

$$\left[\frac{1}{\nu}\left(\frac{\partial\nu}{\partial T}\right)_V\right]_{\text{relativistic}} \approx -(2.28\times10^{-15}) \text{ per } °K, \tag{13}$$

where an unknown uncertainty should be assigned to allow for magnetic specific heat. The agreement between the values in Eqs. (11) and (13) suggests that at 295°K the isomer shift depends explicitly only weakly on temperature. In contrast Benedek and Armstrong[14] found an explicit temperature dependence of $|\psi_\uparrow(0)|^2 - |\psi_\downarrow(0)|^2$, where $\uparrow$ and $\downarrow$ denote spin orientation, from studies of the nuclear magnetic resonance in iron.

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OBSERVATION OF SOLID-STATE PHOTOEMISSION FROM TIN INTO GERMANIUM

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The process of photoemission of electrons from one solid into another in contact with it has been postulated by several workers. Mott and Gurney[1] suggested the existence of an internal photoelectric effect to explain certain data for sodium metal in contact with rock salt crystals. Gilleo[2] investigated the existence of this effect in more recent work with silver on alkali halide crystals. Most recently, Williams and Bube[3] proposed that photoemission from copper and other metals plated on cadmium sulfide crystals takes place. All the above experiments required relatively energetic radiation (greater than 1 ev) to produce effects.

Recently we observed an extended spectral response for photovoltaic and photoconductive effects in tin-plated germanium. The test specimen was a slice of "*n*"-type 30-ohm-cm germanium 20 mils thick, on opposite faces of which were electroplated tin contacts. The contacts were $\frac{1}{4}$ inch$\times\frac{1}{4}$ inch square and were displaced with respect to one another by $\frac{1}{8}$ inch. In the test setup, radiation from a monochromator was made to pass through the germanium and impinge on the tin metal. The photovoltaic or photoconductive signal across the contacts was measured. The radiation was chopped at 450 cps, and the signal was amplified by a narrow-band ac amplifier. Room-temperature measurements on this and many similar specimens failed to show any extended spectral response. However, at liquid-nitrogen and liquid-helium temperatures, an extended spectral response several hundred times smaller than the peak intrinsic responses was detected.

To ensure that intrinsic excitation processes and short-wavelength scattered light did not produce the observed signals, germanium filters of various thicknesses were interposed in the exit beam of the monochromator. Typical results are illustrated in Fig. 1, which shows that the use of filters differing in thickness by a factor of ten caused no significant decrease in long-wavelength

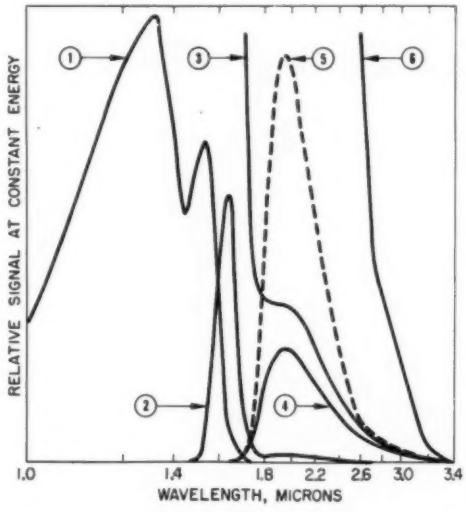

FIG. 1. Photovoltaic signals at constant energy for electroplated tin-on-germanium cell No. 1. Curve 1: the "intrinsic" photovoltaic response with no filtering of the monochromator radiation. Curve 2: radiation is filtered by a 10-mil-thick germanium filter; amplifier gain is ten times that used for curve 1. Curve 3: 10-mil-thick germanium filter is used again, but amplifier gain is ten times that used for curve 2. Curve 4: same gain as that for curve 3, but a 100-mil-thick germanium filter is used. Curves 1 through 4 were taken at liquid-nitrogen temperature. Curve 5: same gain as that for curves 3 and 4, 100-mil-thick filter, but liquid-helium cooled. Curve 6: same conditions as those for curve 5, but amplifier gain increased ten times. The extended spectral response lies at wavelengths greater than 1.8 microns approximately.

response. Two thick filters were used, and the signal response decreased only by a factor attributable to the transmission losses (primarily by reflection) of the filters. Use of these filters demonstrates that intrinsic processes cannot be responsible for the long-wavelength signal. In any case, many measurements of intrinsic absorption coefficients[4] showed no values even approaching those required to produce a measurable signal at the wavelengths involved. Short-wavelength radiation was almost completely eliminated as a source of excitation, and in any case, its intensity would be strongly dependent of filter thickness.

The only other sources of excitation of which the writer can conceive are free-carrier absorption and impurity excitation. The former does not seem capable of producing signals. Furthermore, the absorption coefficient for this process is low, especially at low temperatures where few free carriers are available. Similarly, impurity photoconduction is most improbable. One observes from Fig. 1, curves 3 and 4, that on the short-wavelength side the extended spectral response dropped off rapidly when the thick filter is used (curve 4), but remained approximately constant at 1.8 microns (curve 3) when the thin filter was used. The ratio of the filter transmissions will change rapidly for absorption coefficients in the range 1 to 10 (cm^{-1}); therefore, such values for absorption coefficients are necessary to account for the extended spectral response in terms of an impurity photoconduction process. Neither shallow impurities (which are mostly ionized at liquid-nitrogen temperature) nor deep-lying impurities are present in the near-intrinsic germanium in a concentration sufficient to give an absorption coefficient of one per centimeter. The electroplated tin-on-germanium received no heat treatment so that significant alloying or diffusion could not have occurred. The signal strength depends on the intensity of the light incident on the tin, although these measurements are a little difficult to interpret exactly because of the high reflectivity and refractivity of the germanium which causes much scattering of light within the samples. Finally, the character of the spectral response curves at long wavelengths strongly suggests the photoemission process; one expects a rapid drop in response with an increase in wavelength as a result of the finite mean free paths of excited photoelectrons within the tin cathode, just as for the external photoelectric effect.

Measurements on several tin-plated germanium samples showed that the current-voltage characteristics were variable, although extended spectral responses were obtained in most cases. This variability is not surprising considering that tin is not a specific "dope" for germanium and therefore the exact barriers at contacts may depend on the presence of minute amounts of impurity. Since the current-voltage characteristics were variable, it has been difficult to determine conclusively the effects of applied electric fields on the spectral response. Also, noise resulting from the application of voltage to the cells has frequently interfered with measurements, and the effects of light scattering within samples sometimes obscured results. The

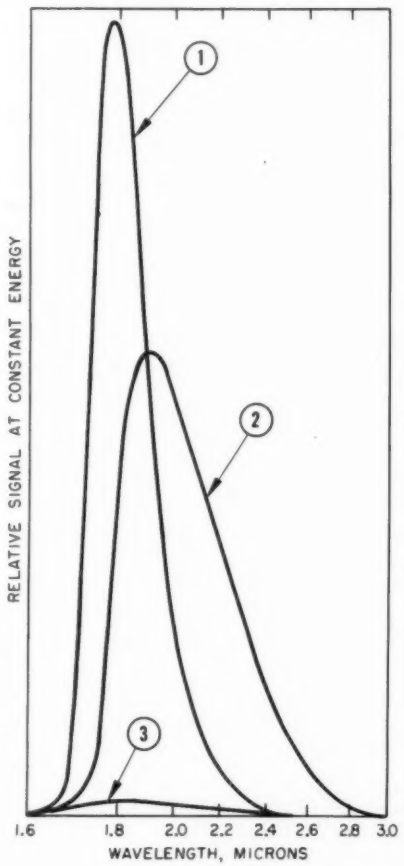

FIG. 2. Photovoltaic and photoconducting signals for electroplated tin-on-germanium cell No. 3 at liquid-nitrogen temperature. Curve 1: photovoltaic signal; amplifier gain is ten times that for curves 2 and 3. Curve 2: photoconductive signal for negative voltage applied to "cathode." Curve 3: photoconductive signal for a positive voltage on the cathode. For all curves, a 100-mil-thick filter was used.

evidence in general, however, supports the expectation that fields which push the electrons away from the metal cathode into the germanium produce larger signals and some extension of the spectral response. Figure 2 shows some particularly good data illustrating these field effects.

With the present electroplated tin-on-germanium specimens, the spectral responses extend to about 3.4 microns. The quantum yields are typically a few tenths of a percent, and significant improvement in these yields may be anticipated. In principle, potential barriers between solids may be very small, and thus photoemission at much longer wavelengths may be ultimately achieved.

The author wishes to thank Dr. G. Picus, Dr. J. M. Richardson, and Dr. J. Grossman for their helpful discussions, Dr. V. Zentner and E. Eccleston for an able job of electroplating, and A. Rabideau for assistance with experimental work.

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NUCLEAR MAGNETIC RESONANCE OF Cr^{53} IN ANTIFERROMAGNETIC $CrCl_3$†

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(Received November 6, 1961)

The metamagnetic properties of the anhydrous iron group chlorides have been the subject of many investigations in the past. Recently, several of these hexagonal layer type compounds were shown to become antiferromagnetic at low temperatures.[1] The magnetic ordering consists of ferromagnetically aligned moments within each hexagonal layer. The antiferromagnetic coupling between adjacent layers is relatively weak and as a consequence steep net magnetization curves are obtained even for small external magnetic fields.[2] This effect suggests that an enhancement mechanism similar to that found in ferromagnets[3] might make it feasible to measure the temperature dependence of the sublattice magnetization in some of these compounds by nuclear magnetic resonance (NMR) techniques.

In this Letter we wish to report the observation of the Cr^{53} NMR in $CrCl_3$ in the temperature range of 1.23°K-4.02°K and zero external field. Results of precision measurements of the frequency for resonance are in excellent agreement with the predictions of simple spin-wave theory for a two-dimensional ferromagnetic system in the presence of anisotropy.

$CrCl_3$ has a hexagonal unit cell, of space group D_3^5, containing 6 molecules.[4] It exhibits a λ-type specific heat anomaly at 16.8°K[5] and a magnetic susceptibility maximum around 20°K. Neutron diffraction studies[6] give an ordered structure in which the axes of magnetization are directed perpendicular to the c axis. Polycrystalline samples for the present study were prepared by vacuum sublimation of the dehydrated salt at 700°C. The nuclear resonance spectra are sensitive to the crystallographic purity of the samples. Resonances in addition to those reported here were observed in samples which exhibited a large number of stacking faults and other imperfections.

The resonance was observed with a frequency-swept FM marginal oscillator.[7] As in the case of ferromagnetic $CrBr_3$,[8] the spectrum consists of a well-resolved triplet due to a nuclear electric quadrupole interaction. From the separation of the satellites, $I=\frac{3}{2}$, and the assumption that the electric field gradient at the Cr positions has cylindrical symmetry about the c axis, we obtain $h^{-1}e^2q_{zz}Q = 0.882 \pm 0.010$ Mc/sec. The central component of the spectrum is shifted by the second order effect to slightly higher frequency (~6 kc/sec) relative to the satellites. This shift agrees in magnitude and direction with the previous assumptions.

A comparison of the intensities of the Cr^{53} resonances in $CrBr_3$ and $CrCl_3$ showed that the enhancement factor in these two compounds is of the same order of magnitude. However, since the quadrupole splitting in $CrCl_3$ is insensitive to the orientation of the magnetization in the hexagonal plane it is not yet certain whether the resonance is excited by domain rotation or domain wall displacement. The observed linewidths are

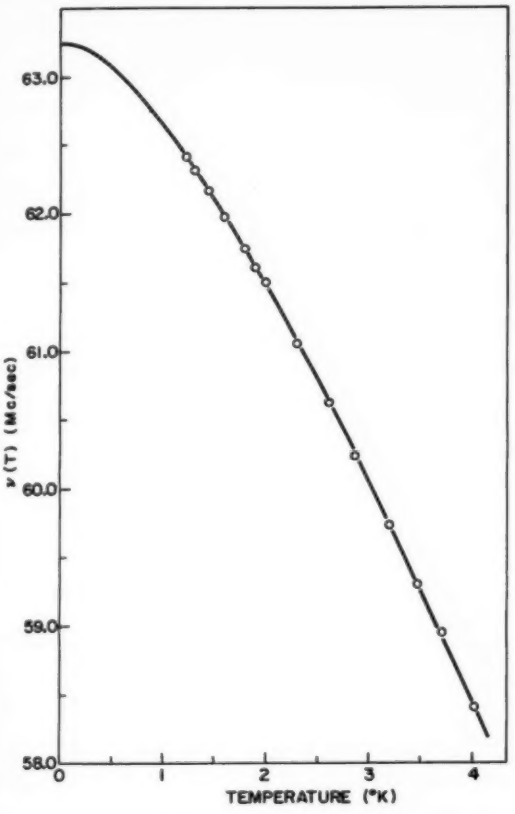

FIG. 1. Temperature dependence of the Cr^{53} NMR (central component) in $CrCl_3$, in zero external field. Solid curve shows the best fit to the data by the two-dimensional spin-wave model.

rather large (~100 kc/sec) and may be caused by a distribution of magnetic dipole fields resulting from imperfect antiferromagnetic order.

In Fig. 1 we show the temperature dependence of the Cr^{53} NMR. If we make the usual assumption that the hyperfine coupling constant does not depend on temperature the observed frequencies are proportional to the sublattice magnetization. The variation of the frequencies is almost linear in temperature and hence cannot be fitted to a T^2 departure ($T_{AE} < T \ll T_N$) or an exponential departure ($T < T_{AE}$) predicted for an antiferromagnet.[9] We can account for our observations by assuming that the antiferromagnetic exchange coupling is sufficiently weak so that its effect can be included in an effective anisotropy field H_A. The total interaction is then of the form $2J\sum_{\text{pairs}}\vec{S}_i\cdot\vec{S}_j + \sum g\beta H_A$, where J is the ferromagnetic intralayer exchange integral. In the long-wavelength limit we calculate for the hexagonal net appropriate to our case:

$$\nu(T) = \nu(0)\left[1 - \frac{\sqrt{3}}{8\pi}\frac{kT}{JS^2}\sum_{n=1}^{\infty}\frac{1}{n}\exp\left(-\frac{ng\beta H_A}{kT}\right)\right].$$

A least squares fit of the data to this form gave $\nu(0) = 63.246$ Mc/sec ($H_{\text{hfs}} = 262\,870$ oe), $H_A = 2700$ oe, and $J = 3.95°$K. The root-mean-square deviation (0.011 Mc/sec) of the data points from the fit is within experimental error. The value of H_{hfs} is about 10% higher than that reported for $CrBr_3$, indicating, as expected, a greater delocalization of the magnetic electrons in $CrBr_3$ than in $CrCl_3$. The effective anisotropy field of 2700 oe is of the same order as the value of the magnetic dipole sum at the Cr sites, suggesting that the antiferromagnetic interlayer coupling is largely of dipolar origin. This view is also supported by the large intensity enhancement observed in this study.

We wish to acknowledge helpful discussions with D. C. Wallace, H. L. Davis, and A. C. Gossard. We also wish to express our thanks to R. G. DeZeeuw for experimental assistance and to J. E. Hesse for preparing some of our samples.

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ANISOTROPY OF THE SUPERCONDUCTING ENERGY GAP IN PURE AND IMPURE TIN

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(Received November 6, 1961)

Anderson's[1] theory of dirty superconductors predicts that the energy gap, which is expected to be anisotropic in pure superconductors, will become isotropic when enough impurity is added that the scattering frequency, $1/\tau$, becomes greater than the energy gap frequency, ω_g. In order to test this prediction experimentally, we have used the far-infrared techniques developed by Richards and Tinkham[2] to investigate the absorptivity of single-crystal surfaces of pure and impure tin over a frequency range spanning the energy gap. Our results give the first spectroscopic evidence for the anisotropy of the superconducting energy gap, and confirm Anderson's ideas about dirty superconductors.

The experiment was performed by observing the far-infrared power reaching a bolometer after it made many reflections from the walls of a nonresonant cavity. Two of the cavity walls were 2.5- by 7-cm slabs of single-crystal tin whose surfaces had been carefully electropolished. The fractional change in power detected by the bolometer when the superconductivity was destroyed by a magnetic field, $(P_S-P_N)/P_N$, was measured over the wavelength range $3\text{ mm}>\lambda>0.5\text{ mm}$ at the temperature $T=0.32\,T_c$. $(P_S-P_N)/P_N$ is of the order of five percent at low frequencies where the superconductor is essentially lossless, and drops to zero at frequencies high enough to cause excitations across the energy gap. The sharp decrease in $(P_S-P_N)/P_N$, which marks the onset of absorption in the superconductor, is a measure of the energy gap.

For our pure samples, estimates based on the theory of the anomalous skin effect indicate that the electrons which are effective in interacting with unpolarized electromagnetic waves of our frequencies (which are higher than the range of validity of the extreme anomalous limit) lie in a band about 30° wide around the Fermi surface, parallel to the surface of the tin, and displaced somewhat from the equator of the Fermi surface. We measure the energy gap averaged over this band so that in the presence of an anisotropic gap, the absorption edge is smeared out.

In Fig. 1, we show the broad absorption edges obtained by measuring three different orientations of pure tin. Because of the large anisotropy within the effective band of electrons, it is not possible to give an accurate gap value for each orientation. Figure 1 does show, however, that the mean gap for the (110) plane is larger than for (001) or (100). This is in qualitative agreement with the anisotropy inferred from ultrasonic attenuation measurements[3,4] which also average the gap over a band around the Fermi surface.

As might be expected from the complicated Fermi surface of tin, the anisotropy of the gap cannot be represented by an ellipsoid. Since tin has two equivalent axes at right angles in the (001) plane, such a model predicts a sharp absorption edge for an (001) surface. In Fig. 1, however, we see that this absorption edge is smeared out even more than those for the (100) and (110) surfaces.

In impure samples with $\omega_g\tau<1$, as a result of the short mean free path, electrons over the whole Fermi surface interact with the electro-

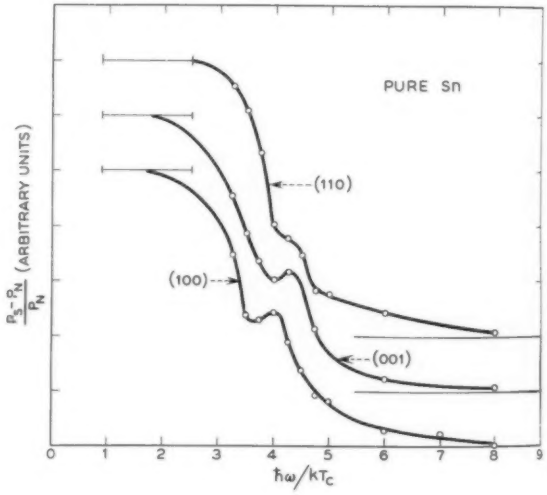

FIG. 1. Frequency dependence, for (110), (001), and (100) planes of pure tin, of the fractional difference between the power reaching the bolometer in the superconducting and normal states. These curves are normalized for display purposes so that the ordinate of the lowest frequency point is the same for all of them, and then displaced to prevent overlapping. The horizontal bar indicates the approximate bandwidth of the lowest frequency point. The bandwidth for the other points was approximately 10%.

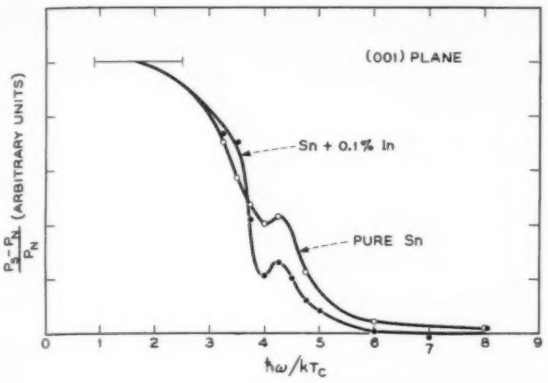

FIG. 2. Frequency dependence, for the (001) plane of pure and impure tin, of the fractional difference between the power reaching the bolometer in the superconducting and normal states. The bandwidths and normalization are the same as in Fig. 1.

magnetic field. Thus, if the gap anisotropy were unaffected in the dirty superconductor, the addition of impurity would smear out the absorption edge even further. The absorption edges for pure and impure (001) tin surfaces in Fig. 2 show, however, that the absorption edge in the impure sample is sharper, not broader than that in the pure sample.

The marked steepening of the absorption edge for tin with 0.1% indium ($\omega_g\tau \approx 0.9$) compared to that for pure tin ($\omega_g\tau \approx 250$) experimentally verifies Anderson's prediction that the energy-gap anisotropy disappears when $\omega_g\tau \lesssim 1$. Other results not shown indicate no further steepening of the absorption edge with added indium and also confirm that the shape and position of the absorption edge for tin with 0.1% indium is essentially independent of crystalline orientation, as would be expected whether or not the gap anisotropy disappeared.

Electron tunneling experiments[5,6] and previous infrared energy gap measurements[2] did not show effects of the anisotropy in tin. However, the sample preparation was such that these were probably experiments on dirty superconductors. We believe that the dirty-superconductor mechanism explains the absence of anisotropy effects in these experiments. We also feel that our results lend considerable support to Masuda's interpretation of his nuclear relaxation experiments.[7]

All of the absorption edges measured show structure in the form of a bump at $\hbar\omega \approx 4.2kT_c$. This structure may be related to that found by Ginsberg, Richards, and Tinkham[8] for lead and mercury. There is one important difference, however, since the tin structure is at a higher frequency than the mean energy gap, while that for lead and mercury is at a lower frequency. At present there is no satisfactory explanation of this structure for any of these metals.

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FLUORESCENCE EXCITATION BY THE ABSORPTION OF TWO CONSECUTIVE PHOTONS*

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(Received November 7, 1961)

Fluorescence radiation from polycrystalline La:$PrCl_3$ has been observed in this laboratory using two separate excitation sources. This fluorescence cannot be attributed to either source alone. The scheme is similar to Bloembergen's[1] proposal for a solid-state infrared quantum counter which was further elaborated upon by this author.[2] Figure 1 shows the partial energy level diagram of Pr^{+++} in a lattice of $LaCl_3$.[3] An infrared signal at 2.3 μ may induce a transition from the $^3H_4$ ground state to the $^3H_6$ state. An electron in the $^3H_6$ state may then be raised to the $^3P_2$ state by the absorption of a photon at ~558 mμ. The $^3P_2$ state is not a resonance level. Electrons in this state decay by a radiationless transition to the $^3P_1$ and the $^3P_0$ states which are resonance levels.[4] The subsequent radiative transition from the $^3P_0$ state to the $^3H_6$ state results in the emission of a photon at ~618 mμ. The electron now in the $^3H_6$ state can be re-excited to the $^3P_2$ state by the incident 558-mμ excitation, thus completing the entire cycle again. If the lattice is at a temperature T such that $kT \ll h\nu$, where $h\nu$ is the energy of the $^3H_6$ level relative to the $^3H_4$ ground state, then this level will be unpopulated and the incident radiation at 558 mμ cannot induce any transitions to the $^3P_2$ state unless there is also an incident signal at 2.3 μ to populate the $^3H_6$ level. Thus the absorption of two consecutive photons at wavelengths 2.3 μ and 558 mμ is necessary to observe fluorescence at 618 mμ.

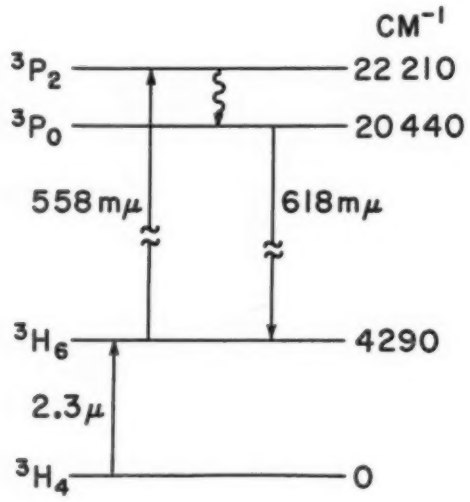

FIG. 1. Partial energy level diagram of Pr^{+++} in a host lattice of $LaCl_3$ showing the only levels involved. The energy given for the $^3P_2$ and $^3H_6$ levels are average energies of the multiplets.

The experimental details are essentially as given in reference 2. The infrared source was a Sylvania DLG projection lamp with Pyrex and germanium filters to give the desired bandpass. The source of the 558-mμ radiation was an AH6 mercury lamp with suitable glass and interference filters. Sharp-cutoff glass filters were used to prevent the direct excitation of the $^3P_2$ state from the $^3H_4$ ground state. An RCA 7265 photomultiplier was used to detect the 618-mμ radiation. Again interference and glass filters were used to prevent the light from the AH6 lamp from entering the

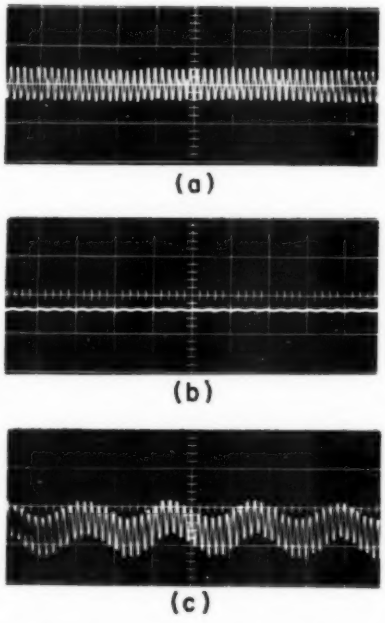

FIG. 2. Oscilloscope trace of phototube output. (a) 558-mμ source alone; (b) 2.3-μ sources alone; (c) 558-mμ and 2.3-μ sources incident on the sample showing the 10-cps modulated component of the output. This is the 619-mμ output modulated by the 10-cps 2.3-μ excitation.

photomultiplier. The infrared source was modulated at 10 cps and the output of the phototube, after amplification, was passed through a low-pass filter and presented on an oscilloscope. The sample was a polycrystalline ingot of 1% $PrCl_3$ in $LaCl_3$ which was refined by vacuum distillation. It was mounted in the tip of a liquid helium Dewar and cooled by conduction.

Figure 2(a) shows the output of the phototube with the 558-mμ excitation alone. The 120-cps signal is the "leak" of the AH6 lamp into the phototube. This is present when the sample is warmed up and is also present in a slightly diminished intensity when the sample in the Dewar tip is replaced by a glass tube similar to the Dewar tip. Figure 2(b) shows the output of the phototube when the infrared source alone is on the sample. Figure 2(c) is the output when both sources are on and clearly shows the 10-cps modulation of the 120-cps signal. This demonstrates that both excitation sources must be on the sample to observe the fluorescence. With a higher degree of isolation between the 558-mμ source and the 619-mμ detector it may be possible to use this solid-state system in the manner proposed by Bloembergen for an infrared quantum counter. This also provides an additional method for studying the lifetimes of the excited states.

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COHERENT LIGHT AMPLIFICATION IN OPTICALLY PUMPED Cs VAPOR*

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An excess population density in the upper of two states connected by an optical transition makes possible light amplification by stimulated emission of radiation (LASER). This has recently been demonstrated in a gaseous system employing collisions of the second kind.[1] We have measured spatially coherent amplification in cesium vapor excited by selective optical pumping.[2]

An optical pump suitable for obtaining a population inversion in cesium is the intense He 3888A line, which overlaps the third cesium resonance line (see Fig. 1). In 1930 Boeckner reported fluorescence in cesium excited by the He 3888A line.[3] We have extended Boeckner's optical-pumping experiment by measuring the absolute intensities of Cs fluorescent lines[4] and from these measurements have computed the populations of the associated states using transition probabilities calculated by the Bates-Damgaard method.[5]

Table I shows the populations of certain energy levels obtained under optimized conditions. The population of the $8P_{1/2}$ energy level was substantially greater than that of the lower levels, $8S_{1/2}$ and $6D_{3/2}$. LASER action was therefore possible at the corresponding wavelengths, $\lambda = 7.2\ \mu$ and $3.2\ \mu$.

The amplification coefficient is related to the populations by the equation,

$$k(\nu) = (h\nu/c)(B_{21}N_2 - B_{12}N_1)S(\nu), \qquad (1)$$

where $S(\nu)$ is the normalized line-shape function $[\int S(\nu)d\nu = 1]$; B is the coefficient of induced emission (Einstein B); N_1 and N_2 are the population densities.

Assuming a Doppler profile, the amplification coefficient at line center reduces to

$$k(\nu_0) = \frac{1}{I}\frac{\partial I}{\partial z} = \left(\frac{\ln 2}{16\, c^2\pi^3}\right)^{1/2} A_{21}\frac{\lambda_0^4}{\Delta\lambda_d}\left(N_2 - \frac{g_2}{g_1}N_1\right), \qquad (2)$$

where A is the spontaneous emission coefficient (Einstein A), g_1 and g_2 are statistical weights, and

Table I. Populations of Cs energy levels under optimized conditions.

Cs energy levels	Population density (10^6 atoms/cm^3)
$8P_{1/2}$	100
$8P_{3/2}$	30
$8S_{1/2}$	10
$6D_{3/2}$	3
$5D_{3/2}$	200

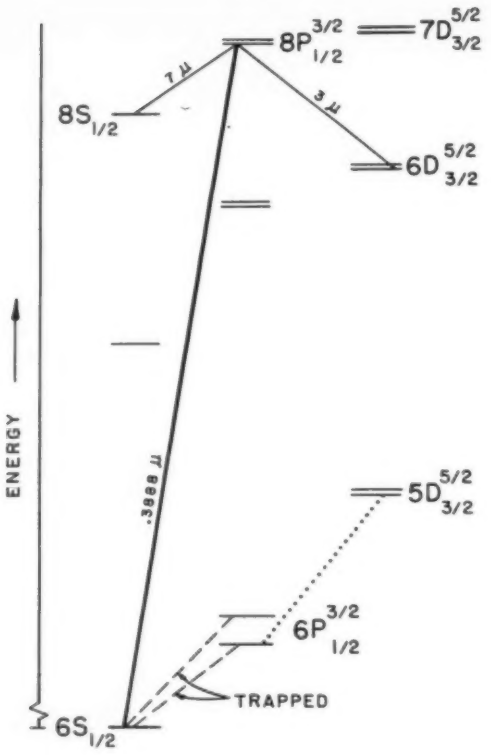

FIG. 1. Cesium doublet diagram showing optical pumping of $8P_{1/2}$ by He 3888A radiation.

$\Delta\lambda_d$ is the Doppler width.

The experimental apparatus for the amplification measurement consisted of an optically pumped Cs fluorescent source, an optically pumped amplification cell, a narrow-band interference filter, and a radiation detector (see Fig. 2). The measurement was carried out at 3.2 μ, rather than at 7.2 μ, because of the superior detectivity of a PbS liquid-nitrogen-cooled detector compared with the best 7μ detectors.

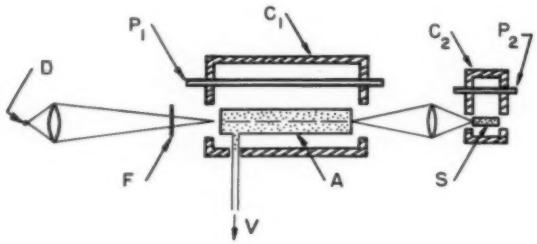

FIG. 2. Experimental arrangement for amplification measurement. P_1, P_2 = helium lamps; C_1, C_2 = reflective cavities; S = Cs source cell, A = Cs amplifier cell, F = narrow bandpass filter, D = radiation detector, and V = entrance to vacuum pump.

The source and amplifier were heated Cs cells, 9 cm and 90 cm long, respectively, positioned within reflective cavities and illuminated by helium discharge lamps. The source lamp, P_2, was operated at 30 Mc/sec and modulated at 105 cps in order to avoid chopping the thermal radiation emitted by the heated Cs cell. The amplifier lamp, P_1, was operated at 400 cps, since rf power, which provides more efficient excitation, was not available to run a tube of its dimensions. The temperature of the Cs was adjusted for maximum fluorescence. The absorption coefficient at the center of the third Cs resonance line was then $k_0 \approx 1/r$, where r is the cell radius. The Cs source and amplifier diameters were 4 and 10 mm, respectively, corresponding to an optimum Cs temperature of 175°C in the source and 165°C in the amplifier. In order to reduce the effects of impurity gases upon the population inversion, a continuously pumped vacuum system was used in the amplifier cell. The conductance was designed to establish a vacuum system gradient of no more than 3% along the amplifier tube in order to insure uniform absorption of pumping light. It was found that fluorescence lines originating from levels ordinarily populated by collisions were reduced greatly in intensity when the vacuum system was utilized.

Phase-sensitive detection was used with a time constant, $\tau \approx \frac{1}{3}$ sec, which made possible detection of 2×10^{-13} watt. The power reaching the $\frac{1}{4}\times\frac{1}{4}$ mm detector from the source was about 10^{-9} watt, and provided a signal-to-noise ratio of 100:1. The discrepancy between system detectivity and observed signal-to-noise was due to mechanical vibration of the very small image with respect to the minute detector.

The direct amplification measurement was made by turning on and off the 400-cps helium lamp illuminating the amplifier cell. The detection system, locked in on 105 cps, showed an increase of (4 ± 1)% each time the amplifier helium lamp was turned on. The following checks were made, each time repeating the above procedure:

1. Cs vapor pressure was reduced; gain disappeared.

2. Source power was turned off; signal went to zero and remained there.

3. Source power was on, but source light was blocked. Signal remained at zero.

4. Wavelength filter was changed to $\lambda = 3.01\ \mu$ ($6P_{1/2} - 5D_{3/2}$ transition); absorption was observed.

5. Linearity of the detection system was checked by introducing $3.2\ \mu$ radiation after, instead of through, the amplifier cell. This was done by means of a beam splitter. The source and amplifier intensities were adjusted so that the power levels reaching the detector were comparable to their original respective values. The detection system was again locked to the source; no enhancement effect was observed.

We feel that the above checks rule out the possibility of artifact. The measured amplification is in reasonable agreement with the value computed from Eq. (2) using populations given in Table I.

$$k(\nu_0) = 0.002\ \text{cm}^{-1} \text{ for } \lambda = 3.2\ \mu.$$

This coefficient may be reduced by a factor as large as 2 due to incipient hyperfine splitting in $8P_{1/2}$. Assuming a Doppler shape for both source and amplifier, the measured amplification coefficient, averaged over the line shape, is smaller by a factor of $\sqrt{2}$ than the value at line center. From these considerations one predicts a measured gain of approximately 6% for a 90-cm cell.

Butaeva and Fabrikant recently investigated the possibility of obtaining a population inversion in Cs by a combination of optical pumping and collisions of the second kind.[6] They measured variations in the relative intensities of the $7D$-$6P$ fluorescence transitions which they suggest may be due to stimulated emission of radiation. The interpretation of Butaeva and Fabrikant rests on the assumption that the lifetimes of $6P_{1/2,3/2}$ are not appreciably lengthened by radiation trapping. According to the theory of light imprisonment due to Holstein,[7] the lifetime of these states should be increased from the natural lifetime of[8] 3×10^{-8} sec to about 10^{-4} sec, under the conditions of their experiment. Since the trapped lifetime is longer than that of any other excited Cs level, we do not believe the populations of $6P_{1/2,3/2}$ could be less than that of any other Cs level, except the ground level. This conclusion we substantiated by measurement of strong absorption in the $6P_{1/2} - 5D_{3/2}$ transition.

An improved apparatus is being built which we believe will increase the amplification coefficient by a factor of 5. An oscillator incorporating Cs vapor is also under construction.

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NEW APPROXIMATION FOR PAIRING FORCES IN NUCLEI*

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It has been observed[1] that a pairing force interaction will account for several features of nuclear structure which were previously unexplained. Numerical calculations have been made[2] for this type of interaction on nuclei having a closed neutron or proton shell, using the quasi-particle approximation. Recently, an exact calculation was done on an electronic computer,[3] which showed that the quasi-particle approximation was not very accurate, but that one could obtain accurate solutions by projecting out that part of the quasi-particle solution corresponding to the correct number of particles. It is the purpose of this note to develop a simple method for solving the pairing force problem, which is as good as the quasi-particle approach in its first approximation and can be improved by an iteration technique to any desired accuracy.

The Hamiltonian which we consider is

$$H = H_{\text{s.p.}} + H_{\text{pairing}}, \tag{1}$$

where

$$H_{\text{s.p.}} = \sum_{\text{unpaired particles}} \epsilon_k a_k^{\dagger} a_k, \tag{2}$$

$$H_{\text{pairing}} = \sum_{\text{pairs}} \epsilon_k (a_k^{\dagger} a_k + a_{-k}^{\dagger} a_{-k}) - G \sum_{p,p'} b_p^{\dagger} b_{p'}. \tag{3}$$

$a_k^{\dagger}(a_k)$ are the usual single-particle creation (annihilation) operators; ϵ_k is the single-particle energy. $b_p^{\dagger}(b_p)$ create (destroy) a pair of particles in the state P. The wave function of a pair is $(-1)^{j-m}|j,m\rangle|j,-m\rangle$ and this phase makes the Hamiltonian, as written here, the same as in references 2 and 3. The pairing interaction, $-G\sum_{p,p'} b_p^{\dagger} b_{p'}$, is understood to take place over a finite energy interval.

Next we define a few quantities to be used in the rest of this note. Let L = number of levels available for occupancy by a pair; P = number of pairs of particles ($\frac{1}{2}$ the number of particles which are paired); N = number of configurations;

$$N = L!/[P!(L-P)!]; \tag{4}$$

E_c = energy of the configuration c;

$\rho_{c,c'} = 0$ if configurations c, c' are not coupled by the pairing interaction,
$= 1$ if configurations c, c' are coupled by the pairing interaction. (5)

To solve our Hamiltonian, we may consider only H pairing and add in the energies of the unpaired particles to the solutions of H pairing, in order to obtain the total energy.

For the pairing Hamiltonian, developing our solution as a sum over configurations, we note

$$C_M[E_M - \lambda - (P-1)G] - G\sum_{M'} \rho_{M,M'} C_{M'} = 0, \tag{6}$$

where C_M is the probability amplitude for the configuration M and λ is an eigenvalue of the pairing Hamiltonian.

Next we set

$$G\sum_{M'} \rho_{M,M'} C_{M'} = \frac{[P(L-P)+1]}{N} G\sum_{M'} C_{M'} + C_M k_M (E_M - \lambda') \tag{7}$$

where

$$\lambda' = \lambda + (P-1)G, \tag{8}$$

and defining

$$G_{\text{eff}} = \left[\frac{P(L-P)+1}{N}\right] G, \tag{9}$$

we may derive the following set of equations:

$$C_M(1-k_M)(E_M - \lambda') = C_{M'}(1-k_{M'})(E_{M'} - \lambda'), \tag{10}$$

$$\frac{k_M}{1-k_M} = G\sum_{M'} \frac{\rho_{M,M'}}{(1-k_{M'})(E_{M'} - \lambda')} - 1, \tag{11}$$

$$\sum_M \frac{k_M}{1-k_M} = 0, \tag{12}$$

$$\sum_M \frac{1}{1-k_M} = N, \tag{13}$$

$$\sum_M \frac{1}{(1-k_M)(E_M - \lambda')} = \frac{1}{G_{\text{eff}}}. \tag{14}$$

Equations (10) through (14) allow us to solve our problem. First we use Eq. (14) to solve for λ', by

setting all $k_M = 0$. Next, we use Eq. (11) to calculate the values of k_M. We put these values of k_M into Eq. (14) and then solve for λ' again. This process may be repeated to obtain any desired accuracy.

It is of interest to examine some special cases which are handled exactly in the first order approximation. If we have only one pair of particles (holes) under consideration, all configurations are coupled by the pairing force, so the exact solution is

$$\sum_M \frac{1}{(E_M - \lambda')} = \frac{1}{G}. \tag{15}$$

This corresponds to nuclei having two or three particles (holes) beyond a closed shell, e.g., Ni^{58}, Ni^{59}, Sn^{130}, and Sn^{129}. Another case which can be solved immediately is when all of the configurations are degenerate in energy. When this is the case, we can extract $(E_M - \lambda')$ from the summation in Eq. (14), and using Eq. (13) we obtain

$$\lambda' = E_M - NG_{\text{eff}}. \tag{16}$$

Looking at Eq. (11), we can see that k_M will be zero for this case.

When the first order calculation is not exact, we have found, in several cases studied, that we can obtain the value of λ' almost exactly (to less than 50-kev error) by using an analytic approximation. We set

$$(1 - k_M) = \alpha_M (E_M - \lambda')^{1/2}. \tag{17}$$

To obtain a first approximation we assume α_M to be a constant, α. This leads to the equation for α:

$$\alpha = \frac{1}{N} \sum_M \frac{1}{(E_M - \lambda')^{1/2}}, \tag{18}$$

and for λ'

$$NG_{\text{eff}} \sum_M \frac{1}{(E_M - \lambda')^{3/2}} = \sum_M \frac{1}{(E_M - \lambda')^{1/2}}. \tag{19}$$

We compute values of α_M using

$$\left(\frac{\alpha}{\alpha_M}\right) = G(E_M - \lambda')^{1/2} \sum_{M'} \left(\frac{\alpha}{\alpha_{M'}}\right) \frac{\rho_{M,M'}}{(E_{M'} - \lambda')^{3/2}}, \tag{20}$$

and obtain improved values of λ', using

$$NG_{\text{eff}} \sum_M \left(\frac{\alpha}{\alpha_M}\right) \frac{1}{(E_M - \lambda')^{3/2}} = \sum_M \frac{1}{(E_M - \lambda')^{1/2}}. \tag{21}$$

Equations (17)-(21) apply only to the lowest eigenvalue of a system of any given seniority and any given set of levels occupied by unpaired particles. For higher eigenvalues, the correction terms $(1 - k_M)$ vary wildly; but the first approximation to λ', obtained by setting $k_M = 0$ in Eq. (14), should not be too bad, as small changes in λ' have large effects on the sum in Eq. (14).

It is of some interest to compare the values of λ', computed by iterating Eq. (14) and Eq. (11) with the value obtained from Eq. (19). We make the comparison for the lowest spin $\frac{11}{2}$ level of Sn^{125}. The parameters of interest are $G = 0.173$ Mev and single hole energies $\epsilon_{(h11/2)} = 0$ Mev, $\epsilon_{(d3/2)} = 0.60$ Mev, and $\epsilon_{(S1/2)} = 0.90$ Mev. Using Eq. (14) and Eq. (11), we obtained a sequence of values for λ' of -1.58, -1.73, -1.79, and -1.81 (all in Mev). Using Eq. (19), we obtained -1.79 Mev. Equation (14), in first order, gives results which are roughly comparable to those of the quasi-particle approximation. Equation (19) cuts down the errors by an order of magnitude. Either equation appears to be easier to solve than the pair of simultaneous equations derived from the quasi-particle approximation.

I would like to thank Dr. Robert Lawson for making available to me a preprint of reference 3 and for a great deal of aid in treating this problem. I also acknowledge several informative discussions with Marshall Luban on the pairing Hamiltonian.

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PERSISTENCE OF ISOBARIC CORRESPONDENCE BETWEEN NUCLEAR STATES IN MASS REGIONS WHERE ISOTOPIC SPIN IS NOT A GOOD QUANTUM NUMBER

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It has been widely assumed that the extent to which states of two isobaric nuclei are similar is determined by the degree of isotopic spin purity in the nuclei. Some recent data[1,2] suggest that this is not correct, and our purpose is to show that this is understandable theoretically.

The data consist of neutron spectra from (p,n) reactions on various nuclei up to mass 93. Each spectrum shows a group at a Q value equal to the Coulomb energy of the added proton. It seems certain that such a group corresponds to the excitation in the final nucleus of a state that is the isobar analog of the target state. Although such isobar states are familiar in nuclei with $A \lesssim 40$, it was expected that such correspondence should not persist when $A \sim 100$.

This expectation can be put on a quantitative basis by considering the extent to which a pure isotopic spin state A is spread in energy among actual states i by the action of Coulomb forces, V_C. The second moment of the spreading:

$$M_2 \equiv \sum_i (E_A - E_i)^2 \langle A|i\rangle^2,$$

where E_A, E_i are energies, is equal to

$$\langle A|(V_C - \bar{V}_C)^2|A\rangle,$$

$\bar{V}_C$ being $\langle A|V_C|A\rangle$. Evaluating this for a Fermi gas, following MacDonald,[3] we find

$$M_2 = (e^4/R^2)Z(Z-1)(0.37+0.017Z),$$

where R is the nuclear radius. For mass 93, $M_2 = 105$ (Mev)^2, suggesting that a pure isotopic spin state is so widely spread as to be unobservable. In contrast, the observed group[2] is less—probably considerably less—than an Mev wide.

We wish to point out that M_2 may be a serious overestimate of the effect we are concerned with. Whereas all Coulomb forces contribute to isotopic spin mixing, the difference between isobaric states of neighboring nuclei arises from the Coulomb interaction of one proton only.

If ψ is the exact ground state of the nucleus with N neutrons and Z protons, the "isobar state" ϕ of the nucleus with $(N-1)$ neutrons and $(Z+1)$ protons is $T_-\psi$, where $T_- \equiv T_1 - iT_2$ is the operator changing neutron into proton. This state will be spread among the actual states of the nucleus by the interaction:

$$v_C \equiv [V_C, T_-]T_-^{-1}.$$

We may compute M_2', the second moment of this spreading:

$$M_2' \equiv \langle\phi|(v_C - \bar{v}_C)^2|\phi\rangle/\langle\phi|\phi\rangle,$$

where $\bar{v}_C = \bar{V}_C$. For a Fermi gas model, ignoring Coulomb space-exchange integrals, we find:

$$R \equiv \frac{M_2'}{M_2} \approx \frac{2N}{Z\delta x}\,\frac{1+0.023Z}{1+0.046Z},$$

where $\delta \equiv N - Z$ and $x \equiv \delta^{-1}(\langle\psi|T^2|\psi\rangle - \frac{1}{4}\delta^2 + \frac{1}{2}\delta)$. Isotopic spin mixing in the state ψ increases $\langle T^2\rangle$ above its minimum value of $\frac{1}{4}\delta(\delta+2)$. If the effective value of T due to mixing is $\frac{1}{2}\delta + y$, then the choices $y = 0,\ 2$ give $x = 1,\ (3+6/\delta)$.

For mass 93, this means $R = 0.15,\ 0.04$, so we see that the correct moment M_2' is considerably less than M_2. The reduction may perhaps be best appreciated by observing that M_2' for mass 93 and $y = 0,\ 2$ equals M_2 for masses 30, 8. This means that the degree of spreading of an isobar state near mass 93 is about the same as that of a pure isotopic spin state in light nuclei.

We have thus shown that correspondence between isobaric nuclear states can persist well beyond the mass region where isotopic spin is a good quantum number.

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EVIDENCE FOR A THREE-PION RESONANCE NEAR 550 Mev*

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A study has been under way of multipion resonances in $\pi^+ + d$ reactions observed in the Lawrence Radiation Laboratory 72-in. bubble chamber exposed to a 1.23-Bev/c pion beam from the Bevatron. A preliminary report on this research was given at the Aix-en-Provence Conference on Elementary Particles[1] where the existence of the ω^0 meson reported by the Berkeley group[2] was confirmed. Since then these data have been substantially increased, although the experiment is still in progress. The existence of a second neutral 3-pion resonance with a mass of approximately 550 Mev is indicated by this larger sample of events.

Many authors[3] have speculated on the existence of neutral, strongly interacting bosons of mass of the order of 3-4 m_π, in order to fit the data for nucleon form factors obtained from electron scattering experiments. These bosons could be readily identified experimentally in the reaction

$$\pi^+ + d \to p + p + X^0. \tag{1}$$

In order to observe the possible decay mode,

$$X^0 \to \pi^+ + \pi^- + \pi^0, \tag{2}$$

we consider the reaction

$$\pi^+ + d \to p + p + \pi^+ + \pi^- + \pi^0. \tag{3}$$

Only events where both protons are visible and at least one proton stops in the chamber with a range less than 15 cm were accepted for analysis.[4]

The events were measured with a digitized microscope and reconstructed by the Berkeley PANG program. A kinematic fit[5] was obtained for the assumed π^0 using the KICK program, and the effective mass of the fitted 3-pion system was then calculated. In order to check the identification of the π^0, we have calculated the missing neutral mass for events which fit our criteria. An ideogram[6] for this missing neutral mass is given in Fig. 1 for the first 199 of our events.

Figure 2 is a histogram of the effective mass of the 3-pion system for our 233 events. An average mass uncertainty on a given event is ~±20 Mev. The large peak near 770 Mev is clearly identifiable as the ω^0. Another large peak in the 3-pion mass plot of Fig. 2 is seen near 550 Mev, which strongly suggests the existence of a second 3-pion resonance (or particle). We shall hereafter refer to this particle as η.

In order to estimate the number of events in this peak which are reasonably due to the η particle, we make the following interpretation of our data. We believe the impulse approximation is reasonably valid because of the loose structure of the deuteron. Thus the basic reaction we are looking at is

$$\pi^+ + n \to p + X^0, \tag{4a}$$

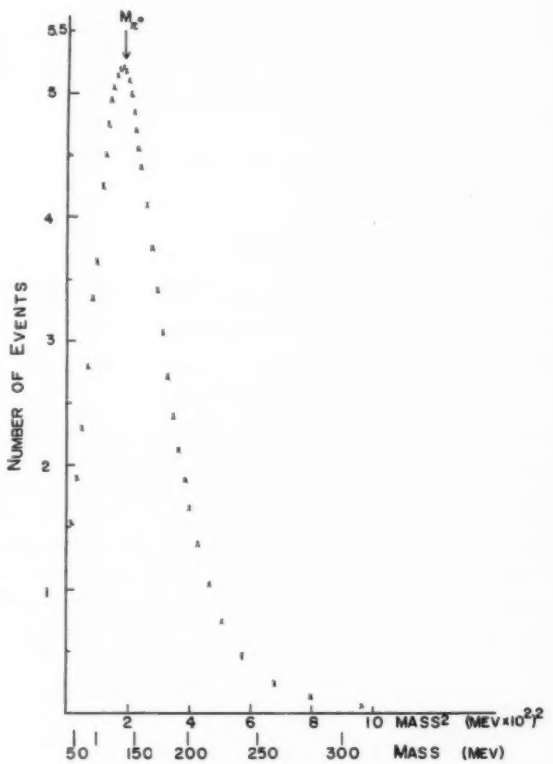

FIG. 1. Ideogram of the missing mass in the reaction ($\pi^+ + d \to p + p + \pi^+ + \pi^-$ + missing mass) for 199 events which meet the selection criteria.

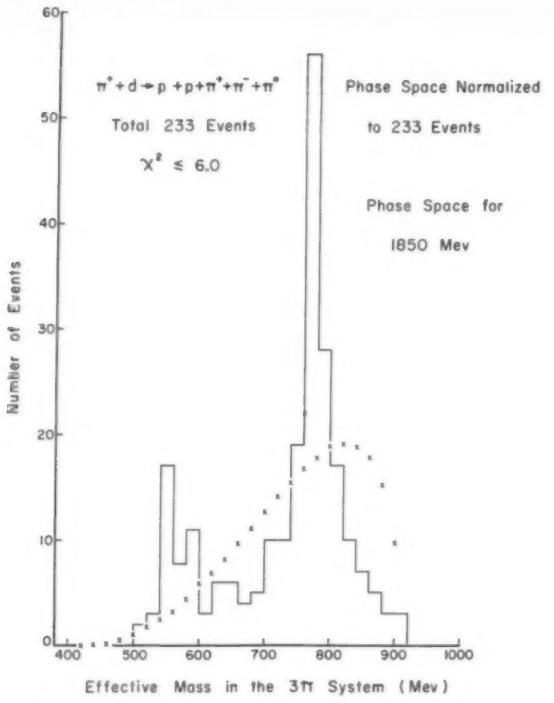
$\pi^+ + d \rightarrow p + p + \pi^+ + \pi^- + \pi^0$
Total 233 Events
$\chi^2 \leq 6.0$
Phase Space Normalized
to 233 Events
Phase Space for
1850 Mev
Number of Events
Effective Mass in the 3π System (Mev)

FIG. 2. Histogram of the effective mass of the three-pion system for 233 events.

where

$$X^0 \rightarrow \pi^+ + \pi^- + \pi^0. \tag{4b}$$

We have calculated the Lorentz-invariant phase space[7] for the 3-pion mass from the background reaction to (4a), i.e.,

$$\pi^+ + n \rightarrow p + \pi^+ + \pi^- + \pi^0, \tag{5}$$

using the experimental average of the total energy in the p-3π center-of-mass system (1850 Mev). This curve, normalized to the total number of events, is plotted in Fig. 2.

Clearly, because of the presence of the ω^0 particle at 770 Mev, such a normalization of phase space yields a gross overestimate of events expected near 550 Mev. Between 540 and 600 Mev there are 36 events in the experimental distribution, whereas the overestimated phase space would account for 12.

An analysis of the data, which takes into account the spread in errors on the individual events on the histogram, gives a mass of approximately 764 Mev with a half-width at half maximum of $\leq$20 Mev for the ω^0 and a mass of ~546 Mev with a half-width at half maximum of $\leq$25 Mev for the η.

An attempt is being made to determine the isotopic spin for both peaks by studing the reaction

$$\pi^+ + d \rightarrow p + \pi^+ + \pi^+ + \pi^- + n. \tag{6}$$

Only 61 events were found in an analysis of one-half the film represented by Fig. 2. The low yield is probably indicative of the lack of any resonance in the isotopic spin states 1 and 2. This is in accord with the Berkeley assignment of $T = 0$ to the ω^0.

A search for the $\pi^0 + \gamma$ decay mode of the ω^0 and η is being carried out by a study of events of the type

$$\pi^+ + d \rightarrow p + p + (\text{neutrals}). \tag{7}$$

The results will be available shortly.

The proton form factor F_{1p} obtained from electron scattering experiments[8] cannot be fitted using only the ω^0 and ρ particles.[9] However, a three-pion resonance of mass $\leq 4m\pi$ having $T = 0$ and spin 1^- would make a fit to the data possible.[10] With the film on hand we expect to more than double our statistics, so that a determination of the isotopic spin and spin of the η may be possible to see whether it fits these theories.

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EXPERIMENTAL STUDY OF THE K_{e3}^+ DECAY INTERACTION*

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Although the V-A theory has been used with considerable success to explain the observed features of the weak decays of nucleons, pions, and muons, only a few experimental tests have been made to determine if the same couplings apply to the leptonic decay modes of strange particles. Insofar as the two-body modes of K^+ mesons are concerned, the experimental upper limit to the K_{e2}^+ ($K^+ \rightarrow e^+ + \nu$) branching ratio (namely about 1%)[1] is certainly compatible with the value of 1.5×10^{-5} expected from the V-A coupling.[2] The interpretation of various features of the K_{e3}^+ ($K^+ \rightarrow e^+ + \pi^0 + \nu$) and $K_{\mu 3}^+$ ($K^+ \rightarrow \mu^+ + \pi^0 + \nu$) decay modes in terms of possible couplings has been the object of considerable theoretical work. Pais and Treiman[3] have pointed out that the K_{e3}^+ mode leads to especially simple predictions and have obtained the following distribution functions of the pion momentum and pion-electron angular correlation:

For vector coupling,

$$F(P,\theta)dPd\cos\theta = \frac{P^4(W^2-P^2)^2 \sin^2\theta\, |f_v|^2}{E(W+P\cos\theta)^4} dPd\cos\theta; \quad (1a)$$

for scalar coupling,

$$F(P,\theta)dPd\cos\theta = \frac{P^2(W^2-P^2)^2 |f_s|^2}{E(W+P\cos\theta)^2} dPd\cos\theta; \quad (1b)$$

for tensor coupling,

$$F(P,\theta)dPd\cos\theta = \frac{P^4(W^2-P^2)^2(P+W\cos\theta)^2 |f_t|^2}{EM^2(W+P\cos\theta)^4} \times dPd\cos\theta, \quad (1c)$$

where P and E are the pion momentum and total energy; M is the K-meson mass; $W = M - E$; θ is the angle between the pion and electron momenta; and the quantities f_v, f_s, and f_t are functions that depend only on the total pion energy, E. The lables vector, scalar, etc., are appropriate to a pseudoscalar K^+ meson and should be replaced by axial vector, pseudoscalar, etc., for a scalar K^+ meson, with no change in the functions (1). Thus the vector, axial-vector coupling leads to the distribution given in Eq. (1a), independently of the K-meson parity. It is clear that for any assumed form factors f_v, f_s, and f_t, the expressions (1) determine the energy spectra and angular correlations of any of the three secondaries

of the decay. Furuichi et al. have calculated the electron energy distributions on the assumption that the energy dependence of the form factors can be neglected,[4] but the experimental data on this point are too meager to allow a conclusive comparison with the theory.[5] Recently Luers et al. have studied the K_2^0 modes, $K_2^0 \to e^{\pm} + \pi^{\mp} + \nu$, in a hydrogen bubble chamber,[6] and have concluded from both the pion energy spectra and the electron energy spectra for fixed pion energy that their data are in good agreement with the vector coupling.

In this Letter we present the results of a study of K_{e3}^+ decay events obtained in an exposure of the 21-liter xenon bubble chamber to a separated K^+ beam at the Bevatron. We identify this decay mode by observing the large radiative energy loss of the charged secondary in the xenon. Our efficiency for recognizing electrons in this way is 90%.[7] In all events used in our analysis, both gamma rays from the π^0 meson convert into electron pairs in the xenon, and these conversion points as well as the K^+ decay points lie within appropriate fiducial volumes in the chamber. With these requirements, we have obtained from the fraction of our film so far processed a sample of 175 events. The quantities measured in each decay are the two gamma-ray directions and the electron direction.

To investigate the pion momentum spectrum, we have studied the distribution of the opening angle ϕ between the two decay gamma rays. This angle is closely correlated with the pion momentum, and its distribution is easily calculated for any given pion energy spectrum. If one neglects the energy dependence of the form factors, the pion momentum distributions appropriate to the three couplings can be obtained by integration of Eq. (1) over θ; these are shown in Fig. 1(a). The corresponding ϕ distributions, calculated with measurement errors of 2 deg folded in, are shown in Fig. 1(b) together with a histogram representing our experimental data.

A χ^2 test shows good agreement between the data and the vector distribution and rules out the other hypotheses with a confidence level of better than 99.9%. We have investigated the possible effects of biases on these conclusions by comparing the expected and observed ϕ distributions for π^0 mesons from the $K_{\pi 2}^+$ ($K^+ \to \pi^+ + \pi^0$) decay mode. This comparison indicates that any biases are small and have no significant effect on the above results.

This test, though very suggestive of the validity of the vector hypothesis, is not absolutely conclusive because of the possibility that a form factor with a strong energy dependence can cause the tensor or scalar couplings to lead to the observed ϕ distribution. This ambiguity can be resolved, at least in principle, by a study of the pion-electron angular correlation.

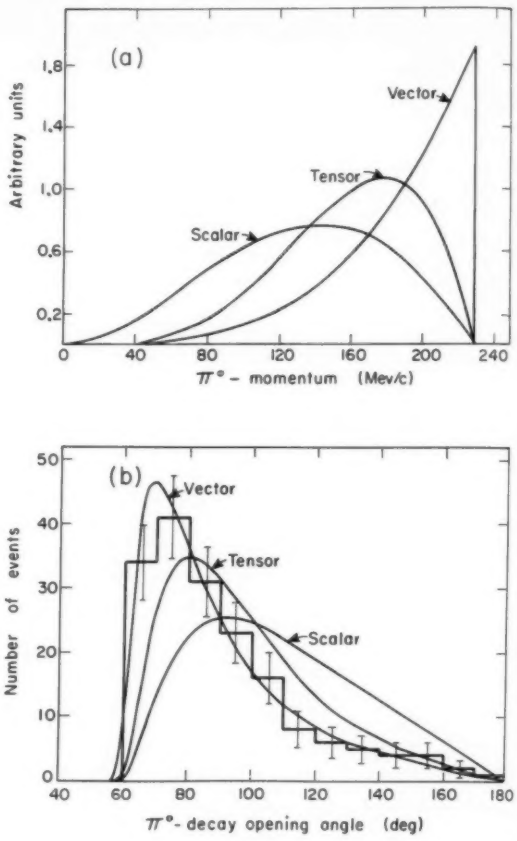

FIG. 1. (a) Pion momentum spectra for the various couplings (constant form factors assumed). (b) Experimental histogram and theoretical distributions (constant form factors assumed) for the π^0-decay opening angle ϕ.

If we require that the predicted π^0 momentum spectrum for each of the three hypotheses agrees with the experimental observations, then the form factors are uniquely determined, at least to the statistical accuracy of the data, and the expected pion-electron angular correlations can be calculated from Eq. (1) by integrating over all π^0 energies. It must be emphasized that the predicted correlations so obtained are independent of a priori assumptions about the energy dependence of

the form factors and, when compared with experiment, provide an absolute test of the validity of each of the possible couplings. We have investigated the pion-electron correlation by studying the distribution of $\cos\theta'$, θ' being the angle between the electron direction and the bisector of the angle ϕ. The direction of this bisector is closely related to the π^0 direction. We have computed the distributions of $\cos\theta'$ expected for each of the various couplings by a Monte Carlo technique. In this calculation the decay configurations were selected in accordance with Eq. (1) and π^0 decay kinematics, with form factors so chosen as to lead to the π^0 momentum spectrum given by the vector curve in Fig. 1(a). This choice of form factors implies a momentum dependence for $|f_S|^2$ ($|f_t|^2$) given by the ratio between the vector and scalar (tensor) curves of Fig. 1(a). This procedure is justified by the excellent agreement between the experimental histogram and the vector curve in Fig. 1(b). The distributions of $\cos\theta'$, predicted by Monte Carlo calculations of 1200 events for each coupling, are shown in the three histograms of Fig. 2. The experimental data, also shown in Fig. 2, are in good agreement with either the vector or the scalar hypothesis, but disagree with the tensor hypothesis with a confidence level of better than 99.9%. The rather poor discrimination between vector and scalar angular correlations arises mainly from the fact that the pion energy distribution is highly peaked at the upper end, in which region the differences in correlations between the various couplings are lessened because of the rapidly varying $(W+P\cos\theta)$ phase-space factors appearing in the denominator of Eq. (1).

If we then take the coupling to be vector, we can set rough limits on the energy dependence of the form factor f_v. If we assume the form $f_v \sim 1+\lambda(q^2/m^2)$, where m is the pion mass and $q^2=M^2+m^2-2ME$ is the square of the four-vector momentum transfer, we obtain with 95% confidence the following limits on λ:

$$-0.05 \leq \lambda \leq 0.25.$$

From the analysis presented in this Letter we draw the following conclusions:

1. The data are consistent with the vector-coupling hypothesis. Within our statistical accuracy, the form factor f_v can be constant or have a variation within the limits quoted above for λ.

2. The data disagree with the tensor-coupling hypothesis for any choice of form factor.

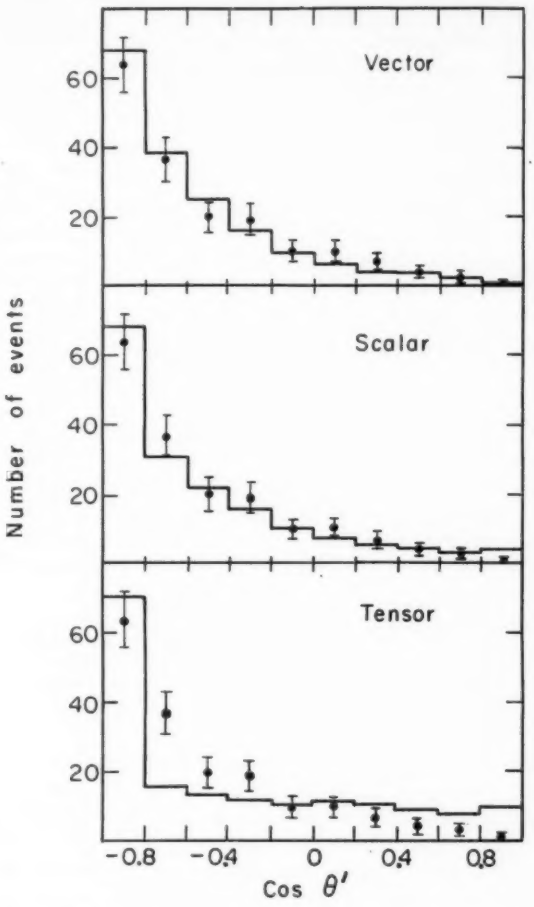

FIG. 2. Comparison between experimental and Monte Carlo $\cos\theta'$ distributions (θ' is the angle between the electron direction and the bisector of the angle ϕ). Varying form factors were chosen as described in text. The histogram represents the Monte Carlo results. The black circles with error flags represent our data.

3. If the scalar form factor is assumed to be slowly varying, the data disagree with the scalar hypothesis. However, we cannot rule out scalar coupling with a highly energy-dependent form factor that is very peaked at the upper end of the pion spectrum.

We wish to thank Professor D. A. Glaser for many helpful discussions connected with this experiment.

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clear Sciences, Brussels, Belgium.

[1]This upper limit of 1% is derived in two ways: (a) In the emulsion data (see reference 5), roughly 20% of the K_{e3}^{+} events have electron energies compatible, within measurement errors, with the value of 247 Mev expected for the K_{e2}^{+} mode. Since the K_{e3} branching ratio is about 5%, the 1% limit follows. (b) In our own branching-ratio study (see reference 7), the observed small number of electron secondaries unaccompanied by electron pairs from π^0 gamma rays leads to an upper limit of less than 1% for the K_{e2} branching ratio.

[2]R. H. Dalitz, Rend. scuola. internaz. fis. "Enrico Fermi" 11, 298 (1960).

[3]A. Pais and S. Treiman, Phys. Rev. 105, 1616 (1957). In Eq. (1) we have considered only pure couplings.

[4]S. Furuichi, T. Kodama, S. Ogawa, Y. Sugahara, A. Wakasa, and M. Yonezawa, Progr. Theoret. Phys. (Kyoto) 17, 89 (1957).

[5]M. Bruin, D. J. Holthuizen, and B. Jongejans, Nuovo cimento 9, 422 (1958).

[6]D. Luers, I. S. Mittra, W. J. Willis, and S. S. Yamamoto, Phys. Rev. Letters 7, 255 (1961).

[7]B. P. Roe, D. Sinclair, J. L. Brown, D. A. Glaser, J. A. Kadyk, and G. H. Trilling, Phys. Rev. Letters 7, 346 (1961).

NEW RESONANCES AND STRONG INTERACTION SYMMETRY*

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Two of the most striking mysteries about the multitude of recently discovered resonances are the following:

(1) The branching ratio for Y_1^*,

$$r=[Y_1^* \to (\pi+\Sigma)_{T=1}]/[Y_1^* \to \pi+\Lambda],$$

seems to be anomalously small, and is consistent with zero. The global symmetry model predicts $r=\frac{1}{2}(p_\Sigma/p_\Lambda)^3=11\%$ whereas the experimental ratio[1] is $4\pm4\%$.

(2) The width of the ω^0 meson ($\to\pi^+ +\pi^- +\pi^0$) is remarkably narrow and consistent with zero. The half-width at half maximum is reported to be less than 12 Mev.[2] (Recall that the total kinetic energy of the decay products is 370 Mev.)

These features once again suggest that there might be a symmetry higher than the symmetry implied by charge independence. We wish to point out that, in a class of theories which are sufficiently symmetric between N and Ξ, it is possible to forbid the $\pi\Sigma$ decay of Y_1^* and make the decay rate for $\omega^0 \to \pi^+ +\pi^- +\pi^0$ vanish to the extent that the $N\Xi$ mass difference could be ignored. Several other consequences of the postulated symmetry are discussed. We also show that the postulated symmetry is a natural consequence of the vector theory of strong interactions provided that N and Ξ (together with Λ and/or Σ) are treated as "fundamental" baryons (not as in the Sakata model).

Let us consider the following discrete operation which we may call hypercharge reflection R.[3]

$$\begin{pmatrix} p \\ n \end{pmatrix} \rightleftarrows \begin{pmatrix} \Xi^- \\ \Xi^0 \end{pmatrix}, \quad \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix} \rightleftarrows \begin{pmatrix} \Sigma^- \\ \Sigma^0 \\ \Sigma^+ \end{pmatrix}, \quad \Lambda \rightleftarrows \Lambda,$$
$$\begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix} \rightleftarrows \begin{pmatrix} \pi^- \\ \pi^0 \\ \pi^+ \end{pmatrix}, \quad \begin{pmatrix} K^+ \\ K^0 \end{pmatrix} \rightleftarrows \begin{pmatrix} K^- \\ \bar{K}^0 \end{pmatrix}. \tag{1}$$

The relative $N\Xi$ parity is assumed to be even, and the spin of the Ξ is assumed to be $\frac{1}{2}$; otherwise speculations given here would be useless. Suppose R is "good" to the extent that the $N\Xi$ mass difference could be ignored. We note that the $\vec{\pi}\cdot(\vec{\bar{\Sigma}}\times\vec{\Sigma})$ interaction, a typical term of which looks like $(\bar{\Sigma}^+\Sigma^+ - \bar{\Sigma}^-\Sigma^-)\pi^0$, must vanish. In contrast the $\vec{\pi}\cdot\vec{\bar{\Lambda}}\vec{\Sigma}$ interaction is fully allowed. Now Y_1^* is known to decay into $\pi+\Lambda$; hence the phenomenological decay interaction $\vec{\pi}\cdot\vec{\bar{\Lambda}}\vec{Y}_1^*$ must be even under R. This immediately implies that Y_1^* must transform like Σ, and that the phenomenological interaction $\vec{\pi}\cdot(\vec{\bar{\Sigma}}\times\vec{Y}_1^*)$ responsible for $Y_1^* \to \pi+\Sigma$ must vanish.

It also follows that R invariance forbids $\pi+\Lambda \neq \pi+\Sigma$, makes the one-pion exchange contribution to $N\Sigma$ forces vanish, and predicts the existence of a $T=\frac{3}{2}$, $p_{3/2}$ $\pi\Xi$ resonance analogous to the πN 3-3 resonance (as in the global symmetry model). The conjectured $\pi\Xi$ resonance may be looked for

in

$$K^- + p \to K + \pi + \Xi.$$

Turning now to the ω meson, one readily sees that if ω is even under R, then $\omega^0 \to \pi^+ + \pi^- + \pi^0$ is "forbidden" to the extent that R is good. [To prove this just note that the phenomenological decay interaction $\omega^0 \vec{\pi}^{(1)} \cdot (\vec{\pi}^{(2)} \times \vec{\pi}^{(3)})$ changes its sign under R.] Thus the narrow width of ω^0 would no longer be a mystery.

The electromagnetic couplings of strongly interacting particles are also invariant under R with $A_\mu \to -A_\mu$ since both the third component of the isospin current and the hypercharge current change their signs. The R invariance of the strong and electromagnetic interactions require that both the Λ^0 magnetic moment (currently being measured) and the Σ^0-Λ^0 transition moment be "zero," and that

$$\mu(\Xi^-) = -\mu(p), \quad \mu(\Xi^0) = -\mu(n), \quad \mu(\Sigma^+) = -\mu(\Sigma^-),$$

where μ stands for the anomalous magnetic moment of the baryon in question. [It also leads to $m(\Sigma^+) = m(\Sigma^-)$ in contradiction with experiments, but this disagreement is not too serious.] Our speculation might go even further: Perhaps the weak interactions of strongly interacting particles are also invariant under R. This immediately leads to the relation

$$\alpha(\Xi^- \to \pi^- + \Lambda) = -\alpha(\Lambda \to \pi^- + p)$$

(where α stands for the asymmetry parameter in Ξ^- or Λ decay), which seems to be satisfied within experimental errors.[4]

One might naturally say: All this is very fine. But is there a "deep" reason to believe in invariance under R any more than in global symmetry or in the more restricted doublet symmetry?[5] It has been conjectured that the Yukawa couplings of pseudoscalar mesons to baryons are actually phenomenological manifestations of the interactions of various vector mesons coupled to the appropriate conserved currents of the strong interactions.[6] Suppose N and Ξ (together with Λ and/or Σ) are introduced as "fundamental" particles in the vector theory of strong interactions (VTSI) of reference 6.[7] The fundamental vector couplings then become invariant under R where the charge triplet vector meson ρ coupled to the isospin current transforms as

$$\begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^- \end{pmatrix} \to -\begin{pmatrix} \rho^- \\ \rho^0 \\ \rho^+ \end{pmatrix}, \qquad (2)$$

and the vector meson coupled to the baryonic current (the hypercharge current) is even (odd) under R.[8] In VTSI, π and K must emerge as bound states of baryons and antibaryons. To the extent that R is "good" for the interactions responsible for the existence of π and K, these pseudoscalar mesons must have definite transformation properties under R. Although it does not follow a priori that π and K must transform with "plus" signs as in (1), the alternative choice with "minus" signs would make the $\vec{\pi} \cdot \vec{\Lambda}\vec{\Sigma}$ coupling vanish, which is highly unlikely in view of our knowledge of ΛN forces.[9]

In a recent note[10] it was conjectured that the observed ω of Maglić et al.[2] is the vector meson of VTSI coupled to the baryonic current, and that the 550-Mev peak in the mass plot for $(\pi^+\pi^-\pi^0)$ of Pevsner et al.[2] is the vector meson η coupled to the hypercharge current. With this identification ω must be even under R, and the narrowness of ω follows. R invariance does not explain why η is narrow, but perhaps the smaller phase space available and the weaker $\eta\bar{N}N$ coupling makes it narrow. We can also see that R invariance "forbids" "isoscalar photon" $\to \omega^0$. It is gratifying that a double Clementel-Villi type fit[10] to the isoscalar charge form factor with two poles (at $m_\eta{}^2$ and $m_\omega{}^2$) does not require too large a value for the residue of the ω pole despite the very strong coupling of ω to $\bar{N}N$. As for the "electromagnetic decays" of ω, note that $\omega^0 \to \pi^0 + \gamma$ and $\omega^0 \to 2\pi^0 + \gamma$ are "forbidden"; $\omega^0 \to \pi^+ + \pi^- + \gamma$ is "allowed" only if the two pions are in an odd relative l state. In contrast, $\eta^0 \to \pi^0 + \gamma$ and $\eta^0 \to 2\pi^0 + \gamma$ are "allowed"; $\eta^0 \to \pi^+ + \pi^- + \gamma$ is "allowed" only if the two pions are in an even relative l state.

So far our statements have been completely independent of the $\Lambda\Sigma$ parity and the dynamical origin of $Y_1{}^*$. Since there seems to be some difficulty in interpreting $Y_1{}^*$ as a resonance of the Dalitz-Tuan type,[11] it is tempting to consider $Y_1{}^*$ as an object analogous to the 3-3 resonance with even $\Lambda\Sigma$ parity.[12] Amati, Stanghellini, and Vitale[13] emphasized that $Y_1{}^*$ could emerge as a $p_{3/2}$ $\pi\Lambda$ resonance not only in the global symmetry model but also in a wide class of models in which the $\Lambda\Sigma$ parity is even, and the $\pi\Lambda\Sigma$ coupling is strong. Indeed with

$$f_{\pi\Lambda\Sigma}{}^2 \approx f_{\pi NN}{}^2 \gg f_{\pi\Sigma\Sigma}{}^2 \approx 0,$$

Franklin[14] was able to show (using the Chew-Low type method) that: (1) it is possible to accommodate $Y_0{}^*$, $Y_1{}^*$, and $Y_2{}^*$ ($T = 2$ resonance sometimes denoted by Z^*) as $p_{3/2}$ resonances; and

(2) $Y_1^* \to \pi + \Sigma$ is forbidden. Our considerations show that his conclusion (2) can be regarded as a consequence of R invariance which forbids both the $\pi\Sigma\Sigma$ coupling and the $\pi\Sigma Y_1^*$ coupling.

Gell-Mann[3] has considered a unitary symmetry model of strong interactions, commonly called the "eightfold way," in which N, Λ, Σ, and Ξ form an octet. Such a model can also accommodate R invariance in a rather natural manner.[15] Assuming that Y_0^*, Y_1^*, and Y_2^* are all $p_{3/2}$ resonances as in Franklin's theory, we might naturally ask: To what representation of the eightfold way do they belong? The answer is that they belong to a "representation 27" including a $T = \frac{3}{2}$ $K\Sigma$ resonance (which would be the "same thing" as the πN 3-3 resonance in the unitary symmetry limit), a $T = \frac{3}{2}$ $\pi\Xi$ resonance, a $T = 1$ KN resonance (which does not seem to exist[16]), and a host of others.[17] Detailed considerations along this line will appear elsewhere.[18]

It goes without saying that the very existence of a partial symmetry, broken within the realm of the strong interactions, is in manifest contradiction with the idea of Chew and others[19] that all strong interactions are as strong as possible.

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*Work supported by the U. S. Atomic Energy Commission.

[1]P. Bastien, M. Ferro-Luzzi, and A. H. Rosenfeld, Phys. Rev. Letters 6, 702 (1961), summarize the current experimental data on the Σ/Λ ratio.

[2]B. C. Maglić, L. W. Alvarez, A. H. Rosenfeld, and M. L. Stevenson, Phys. Rev. Letters 7, 178 (1961); N. H. Xuong and G. R. Lynch, Phys. Rev. Letters 7, 327 (1961); A. Pevsner in the Proceedings of the Aix-en-Provence International Conference on Elementary Particles, September, 1961 (to be published); M. L. Stevenson, L. W. Alvarez, B. C. Maglić, and A. H. Rosenfeld, Phys. Rev. (to be published).

[3]This operation is essentially the same as R in M. Gell-Mann, "The Eightfold Way: A Theory of Strong Interaction Symmetry," California Institute of Technology Scientific Laboratory Report CTSL-20 (unpublished). This paper of Gell-Mann should not be confused with his more recent work, "Symmetries of Mesons and Baryons" [Phys. Rev. (to be published)], which makes a much smaller number of "predictions."

[4]W. B. Fowler, R. W. Birge, P. Eberhard, R. Ely, M. L. Good, W. M. Powell, and H. K. Ticho, Phys. Rev. Letters 6, 134 (1961) give $\alpha_\Lambda \alpha_\Xi = -0.65 \pm 0.35$ while the most likely value of α_Λ seems to lie between -0.7 and -0.85 (private communication from R. H. Dalitz based on work of several groups). This disagrees with "weak global symmetry" of S. B. Treiman [Nuovo cimento 15, 916 (1960)] and others which requires $\alpha_\Xi = +\alpha_\Lambda$. More recently, however, A. Pais [Phys. Rev. 122, 317 (1961)] seems to commit himself to only the weaker condition $|\alpha_\Xi| = |\alpha_\Lambda|$.

[5]M. Gell-Mann, Phys. Rev. 106, 1296 (1957); J. Schwinger, Ann. Phys. 2, 407 (1957); A. Pais, Phys. Rev. 110, 574 (1958).

[6]J. J. Sakurai, Ann. Phys. 11, 1 (1960).

[7]We would like to emphasize that R invariance cannot be accommodated in a theory based on the Sakata model in which only p, n, and Λ are "fundamental." In particular the vector meson theory of A. Salam and J. C. Ward [Nuovo cimento 20, 419 (1961)] is not invariant under R.

[8]The R invariance of the vector couplings also holds in the more generalized vector meson theory of Gell-Mann[3] in which a strange vector meson (K^* ?) is coupled to a quasi-conserved strangeness-changing current.

[9]See, e.g., F. Ferrari and L. Fonda, Nuovo cimento 9, 842 (1958); R. H. Dalitz, in Proceedings of the Rutherford Jubilee International Conference at Manchester, September, 1961 (to be published).

[10]J. J. Sakurai, Phys. Rev. Letters 7, 355 (1961).

[11]W. E. Humphrey, Ph.D. thesis, University of California Radiation Laboratory Report UCRL-9752 (unpublished), and R. R. Ross, Ph.D. thesis, University of California Radiation Laboratory Report UCRL-9749 (unpublished). It is a pleasure to thank Professor R. H. Dalitz and Professor S. F. Tuan for enlightening discussions on the interpretations of Y_1^*.

[12]Nearly all arguments in favor of odd $\Lambda\Sigma$ parity given in Y. Nambu and J. J. Sakurai [Phys. Rev. Letters 6, 377 (1961)] are either wrong or obsolete. The author believes that the following parity combination is more likely to be the correct one: $\Lambda\Sigma$ even, $K\Lambda$ odd, and $N\Xi$ even. This agrees with Gell-Mann's eightfold way.[3]

[13]D. Amati, A. Stanghellini, and B. Vitale, Nuovo cimento 13, 1143 (1959); Phys. Rev. Letters 5, 524 (1960).

[14]J. Franklin (to be published).

[15]In Gell-Mann's language,[3] we must consider couplings of pseudoscalar mesons of the "D type." S. Coleman and S. L. Glashow [Phys. Rev. Letters 6, 423 (1961)] show that R invariance in the eightfold way is incompatible with observation. It is easy to see, however, that most statements made in their paper are expected to be accurate only up to a factor of $(m_K/m_\pi)^2 \approx 13$.

[16]Perhaps the exchanges of ρ and η between K and N make the $T = 1$ KN interaction so repulsive (as conjectured in reference 6) that the attraction in the $p_{3/2}$ state due to the Yukawa-type $K\Lambda N$, $K\Sigma N$ couplings gets cancelled to such an extent that no $p_{3/2}$ resonance is possible.

[17]Should most of the $p_{3/2}$ resonances of the representation 27 be observed, we might regard Y_1^* and Y_2^* as "eightfold way resonances" (or "twenty-seven-fold way resonances") rather than as "global symmetry resonances."

[18]S. L. Glashow and J. J. Sakurai, Nuovo cimento (to be published).

[19]G. F. Chew and S. C. Frautschi, Phys. Rev. Letters 5, 580 (1960); Phys. Rev. 123, 1478 (1961).

PRODUCTION CROSS SECTION OF INTERMEDIATE BOSONS BY NEUTRINOS IN THE COULOMB FIELD OF PROTONS AND IRON

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This is a report on a numerical computation of the cross sections for the processes[1]

$$\nu' + Z \to W^+ + \mu^- + (Z \text{ in various states}), \quad (1)$$

and

$$\bar{\nu}' + Z \to W^- + \mu^+ + (Z \text{ in various states}), \quad (2)$$

where W is the (proposed) intermediate boson for weak interactions, ν' is the neutrino associated with the μ meson, and Z represents a target proton or nucleus. Pion production is not included. The two processes (1) and (2) have the same differential cross sections because of the following theorem:

Theorem 1. For process (2) consider a mirror image of process (1) with all momenta and helicities (i.e., polarization along direction of motion) of the corresponding particles reversed. The differential cross sections to the lowest order in e and g are identical.

To prove the theorem we perform the operation CR (where C = charge conjugation and R = space inversion) on the leptons and W, and the operation R on the nucleus. The theorem follows immediately.

Another theorem that is useful is:

Theorem 2. Consider reaction (1). The differential cross sections to the lowest order in e and g are unchanged under a mirror reflection of all momenta, if the helicities are held fixed.

To prove this theorem we perform a time-reversal operation and prove that the matrix element changes into its complex conjugate.

The interaction Lagrangian of the W with the leptons is taken to be[1]

$$\mathcal{L}_{\text{int}} = ig\psi_\mu{}^\dagger \gamma_4 \gamma_\lambda (1+\gamma_5)\psi_{\nu'}\phi_\lambda{}^* + \text{Hermitian conjugate}, \quad (3)$$

where ϕ_λ, ψ_μ, and $\psi_{\nu'}$ denote the fields describing W^+, μ^-, and ν'. The magnitude of g is[1] given by

$$g = m_W G_V{}^{1/2} 2^{-1/4}, \quad G_V \cong 10^{-5} m_p{}^{-2}. \quad (4)$$

The Lagrangian of the W particle and its interaction with the electromagnetic field A_μ is taken to be

$$\mathcal{L}_{W-A} = -\tfrac{1}{2}(\partial A_\mu/\partial x_\nu)^2 - \tfrac{1}{2} G_{\mu\nu}{}^* G_{\mu\nu} - m_W{}^2 \phi_\mu{}^* \phi_\mu - ie\kappa F_{\mu\nu} \phi_\mu{}^* \phi_\nu, \quad (5)$$

where

$$F_{\mu\nu} = \partial A_\nu/\partial x_\mu - \partial A_\mu/\partial x_\nu,$$

$$G_{\mu\nu} = \partial_\mu \phi_\nu - \partial_\nu \phi_\mu,$$

$$\partial_\mu = \partial/\partial x_\mu - ieA_\mu, \quad (6)$$

$$e = (4\pi/137)^{1/2}. \quad (7)$$

The parameter κ is related to the magnetic moment of the W^+ (along its spin):

$$(e\hbar/2mc)(1+\kappa). \quad (8)$$

There are two Feynman diagrams that contribute in the lowest order to (1), as illustrated in Fig. 1,

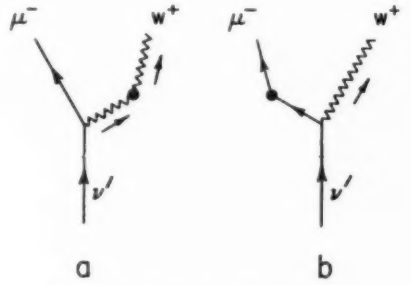

FIG. 1. Feynman diagrams.

in which the black circle represents the electromagnetic field of the nucleus. It contributes a factor V_β to the process. For the coherent process the nucleus recoils as a whole and we take

$$V_4 = \frac{iZe}{q^2} F^Z(q^2), \quad V_1 = V_2 = V_3 = 0, \tag{9}$$

where Ze = nuclear charge, and[1] $q = p' - p$. The form factor F^Z is taken to be

$$F^Z = (1 + \tfrac{1}{12} q^2 a^2)^{-2}, \tag{10}$$

with

$$a^2 = \tfrac{3}{5}\{1.3 \times 10^{-13} A^{1/3}\}^2 \text{ cm}^2. \tag{11}$$

For incoherent processes on nuclei or for production from a free proton, we take

$$V_\beta = (ie/q^2) u_{p'}^\dagger \gamma_4 \times \{F_1 \gamma_\beta + iF_2 K \tfrac{1}{2}(\gamma_\lambda \gamma_\beta - \gamma_\beta \gamma_\lambda)(p - p')_\lambda\} u_p, \tag{12}$$

where $u_{p'}$ and u_p are the Dirac spin wave functions of the final and initial protons, normalized so that

$$u_{p'}^\dagger u_{p'} = u_p^\dagger u_p = 1, \tag{13}$$

$$K = \tfrac{1}{2}(\mu_p) = 0.8948,$$

and F_1 and F_2 are the form factors of the proton which we take from the Stanford experiments.[2]

The differential cross section for (1) is easily shown to be

$$d\sigma = (32\pi^5)^{-1} |\alpha_a + \alpha_b|^2 \times d^3\mu d^3W \, \delta(E_\mu + E_W + E_{p'} - E_\nu - E_p), \tag{14}$$

where α_a and α_b are the contributions to the matrix element from diagrams a and b. They are

Table I. Total cross section in 10^{-38} cm^2 for $\nu' + Z \to W^+ + \mu^- + \ldots$, for Z = proton and Z = Fe.

m_W	κ	E_ν/m_p	σ_p	$\frac{1}{56}\sigma_{\text{Fe}}$(coherent)	$\frac{1}{56}\sigma_{\text{Fe}}$(total)
$0.6m_p$	0	1	0.00564	...	0.006
		1.5	1.39	0.393	1.77
		1.8	2.95	1.23	4.13
		2	4.26	2.17	6.35
		2.5	8.01	6.35	14.12
		3	11.9	13.09	24.49
		4	19.8	35.03	53.48
		5	27.4	66.33	91.18
		6		104.6	
		8		192.3	
		10		287.4	
$1\ m_p$	0	2	0.0623	0.00739	0.0694
		2.5	0.608	0.0507	0.657
		3	1.66	0.202	1.85
		3.5	3.1	0.57	3.65
		4	4.88	1.20	6.03
		5	8.78	3.76	12.40
		6	13.0	8.2	20.88
		7		15.57	
$1\ m_p$	1	3	2.44	0.233	2.66
		4	7.18	1.41	8.54
		5	13.48	4.51	17.81
$1\ m_p$	-1	3	1.30	0.182	1.48
		4	3.77	1.05	4.78
		5	6.88	3.25	10.01
$1.4m_p$	0	3	0.0262	0.00217	0.0283
		4	0.612	0.0269	0.637
		5	1.99	0.139	2.12
		6	3.91	0.4	4.30
		7		1.12	

$$a_a = -2egi\{(\nu-\mu)^2+m_W^2\}^{-1}$$

$$\times\{2[l\phi][WV]+[LV][q\phi]-[Lq][\phi V]\},$$

and

$$a_b = 2egi\{\nu-W)^2+m_\mu^2\}^{-1}$$

$$\times\{2[l\phi][\mu V]+[lV][q\phi]-[lq][\phi V]+\det\}, \quad (15)$$

where

$$L = -(1+\kappa)l-[l\mu]\{(1-\kappa)W-\kappa q\}m_W^{-2}; \quad (16)$$

ϕ = amplitude of W:

$(2E_W)^{1/2}\vec{\phi}$ = unit vector $\perp \vec{W}$ for transverse W

$= E_W(m_W)^{-1}$ (unit vector $\parallel \vec{W}$) for longitudinal W,

$(2E_W)^{1/2}\phi_4 = 0$ for transverse W

$= i|\vec{W}|(m_W)^{-1}$ for longitudinal W. (17)

The four-vector l is defined to be

$$l_\lambda = u_\mu^\dagger \gamma_4\gamma_\lambda u_\nu, \quad (18)$$

where u_μ and u_ν are the Dirac spin wave functions of the μ^- and ν' normalized like (13). The symbol det in (15) is the determinant formed by the components of ϕ, V, l, and q:

$$\det = \epsilon_{\xi\eta\zeta\lambda}\phi_\xi V_\eta l_\zeta q_\lambda, \quad (19)$$

where ϵ is the completely antisymmetrical tensor.

The calculation was made with an IBM-7090 computer. To maximize the flexibility of the code used, the calculation was made separately for each of the two polarization states of the μ, the three polarization states of the W, and the different spin arrangements of the proton. The differential cross sections were computed and various partial sums were taken. The total 7090 time used was of the order of 10 hours. In Table I we present the results obtained for the total cross sections for the production process (1) on protons and on Fe. The results are plotted in Fig. 2. Sample energy spectra are given in Fig. 3.

The total coherent cross section for (1) is denoted by σ_Z(coherent), and the total cross section for $\mu^- + W^+$ production on a proton is denoted by σ_p. (In none of these are pion production processes included.) The sum total cross section σ_Z(total) including both coherent and incoherent processes

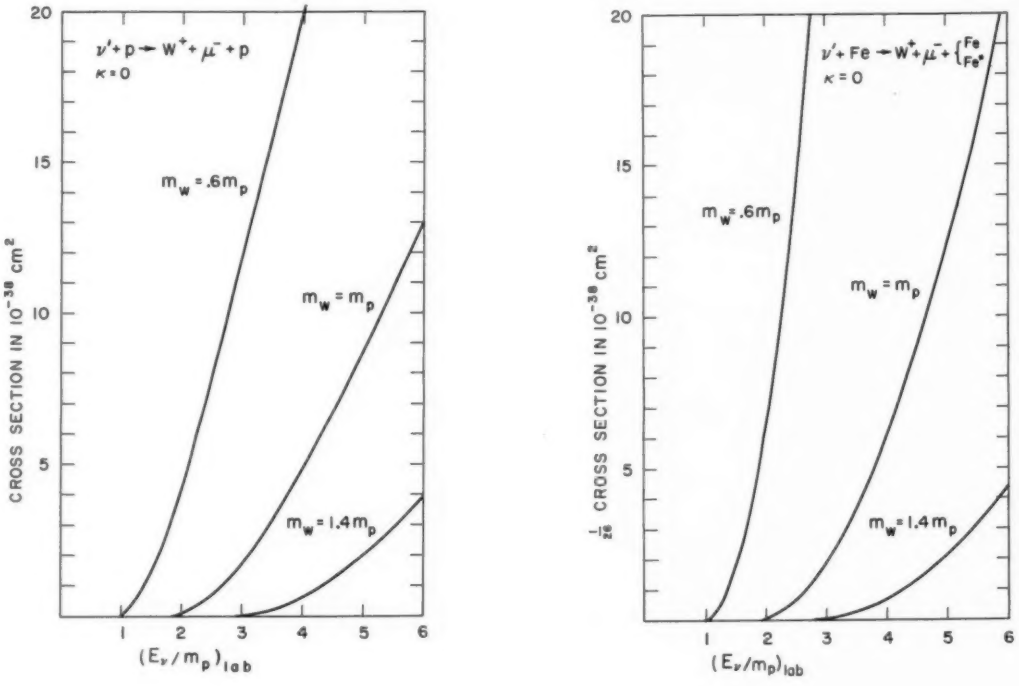

FIG. 2. Total production cross sections for process (1), (a) for protons; (b) for iron.

is computed from

$$\sigma_Z(\text{total}) \cong Z\sigma_p + (1-1/Z)\sigma_Z(\text{coherent}). \quad (20)$$

In this expression the terms $Z\sigma_p$ and σ_Z(coherent) are obvious contributions. The correction term $(-1/Z)\sigma_Z$(coherent) is to subtract out the contributions from those incoherent processes, included in $Z\sigma_p$, which give rise to small momentum transfers, and which therefore are prohibited as incoherent processes. (20) is essentially the same as the corresponding formula used by Masek, Lazarus, and Panofsky[3] for photoproduction of μ pairs.

As is evident from Table I, for high-energy ν', the contribution from the coherent process dominates. The reason for this is well known: For higher energies, the minimum momentum transfer becomes smaller, and the coherent process becomes more effective.

An examination of the energy distribution of the μ and W shows that in general the W carries most of the energy. Consequently the decay μ from the W is in general more energetic than the μ produced in company of the W.

For completeness, we list some asymptotic formulas. The total coherent cross section has been given before,[4,5]

$$\sigma_Z(\text{coherent}) \to \left(\frac{Z}{137}\right)^2 \frac{g^2}{6\pi m_W{}^2}(\kappa-1)^2(\ln\xi)^3, \quad (21)$$

where

$$\ln\xi = \ln[2(\sqrt{12})E_\nu/m_W{}^2 a] \gg 1.$$

This asymptotic formula is, however, only applicable for E_ν much larger than those listed in Table I.

The asymptotic formula for the spectrum in coherent production is

$$\frac{d\sigma_Z(\text{coherent})}{dE_W} \to \left(\frac{Z}{137}\right)^2 \frac{g^2}{2\pi m_W{}^2}\frac{1}{E_W}\left(\ln\frac{2(\sqrt{12})E_W}{m_W{}^2 a}\right)^2$$

$$\times[\tfrac{1}{2}E_W{}^2 - E_W E_\nu + E_\nu{}^2]E_\nu{}^{-2}, \quad (22)$$

which is also applicable only at very high energies.

Two of the authors (T.D.L. and C.N.Y.) take this opportunity to thank Dr. H. Goldstine for permission to use the facilities of the IBM Research Center for the numerical computation described in this Letter.

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[1]We choose units so that $\hbar = c = 1$ and use the notation of T. D. Lee and C. N. Yang, Phys. Rev. 119, 1410 (1960). A four-momentum has real space components and pure imaginary fourth component. A four-vector squared is defined so that $p^2 \equiv p_1{}^2 + p_2{}^2 + p_3{}^2 + p_4{}^2$. All quantities are in the laboratory system. $\vec{\nu}$, $\vec{p}$, $\vec{p}'$, $\vec{W}$, and $\vec{\mu}$ denote the 3-momenta of the ν', the initial and

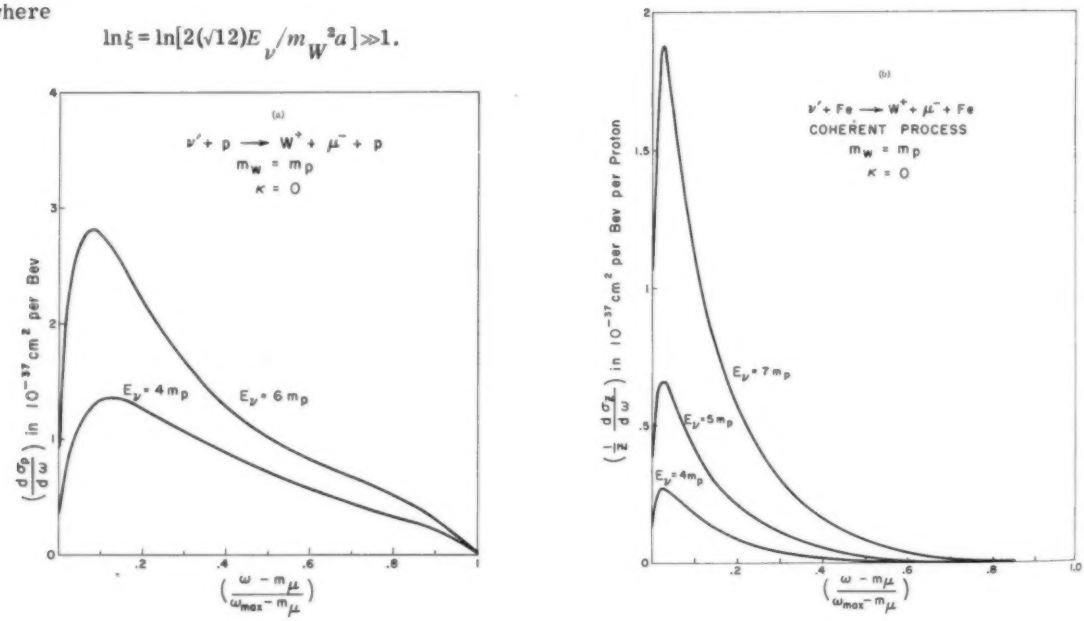
(a)
ν' + p → W⁺ + μ⁻ + p
m_W = m_p
κ = 0
E_ν = 4 m_p
E_ν = 6 m_p
(dσ_p/dω) in 10⁻³⁷ cm² per Bev
(ω − m_μ)/(ω_max − m_μ)
(b)
ν' + Fe → W⁺ + μ⁻ + Fe
COHERENT PROCESS
m_W = m_p
κ = 0
E_ν = 7 m_p
E_ν = 5 m_p
E_ν = 4 m_p
(1/Z²)(dσ_Z/dω) in 10⁻³⁷ cm² per Bev per Proton
(ω − m_μ)/(ω_max − m_μ)

FIG. 3. Energy spectrum of the μ, (a) for protons: (b) for iron. ω = total energy of μ in the center-of-mass system. E_ν = energy of ν' in the laboratory system.

final nucleus (or proton), the W, and the μ. E_ν, E_p, $E_{p'}$, E_W, and E_μ denote their respective energies, and ν, p, p', W, and μ their respective 4-momenta. The symbol $[pq]$ denotes the 4-product of p and q. m_W, m_p, and m_μ are the masses of W, p, and μ.

[2]R. Hofstadter, F. Bumiller, and M. Croissiaux, Phys. Rev. Letters 5, 263 (1960). For momentum transfer $q^2 \geq 25$ (fermi)$^{-2}$ we use the arbitrary extrapolation: $F_1 = 0.4$ and $F_2 = 0$.

[3]G. E. Masek, A. J. Lazarus, and W. K. H. Panofsky, Phys. Rev. 103, 374 (1956).

[4]T. D. Lee and C. N. Yang, Phys. Rev. Letters 4, 307 (1960).

[5]T. D. Lee, Proceedings of the 1960 Conference on High-Energy Physics at Rochester (Interscience Publishers, Inc., New York, 1960), p. 566.

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EVIDENCE FOR THE DOUBLE F MODEL OF THE M CENTER, Bruce J. Faraday, Herbert Rabin, and W. Dale Compton [Phys. Rev. Letters 7, 57 (1961)].

F. A. Kröger has kindly pointed out an error of a factor of two in the computations. At low temperature the constant K should be taken as 6 rather than 12 owing to the symmetry of the M center. This correction gives the following results:

	f_F/f_M	f_M
KBr (He temp.)	1.6	0.30
KBr (N_2 temp.)	2.0	0.24
KCl (He temp.)	1.4	0.40
KCl (N_2 temp.)	1.5	0.36

These values are now in reasonable agreement with Okura's determination (reference 9). Similarly the constant K for NaCl and LiF at room temperature becomes 3500 and 2200, respectively. Values of r/a become 5.9 for NaCl and 5.1 for LiF.

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